PLAIN TRIGONOMETRY

Rendered *Easy* and *Familiar*,

By CALCULATIONS in ARITHMETICK only:

WITH ITS

APPLICATION and USE

In ascertaining all Kinds of HEIGHTS, DEPTHS, and DISTANCES,

IN THE

HEAVENS, as well as on the EARTH and SEAS;

WHETHER OF

Towers, Forts, Trees, Pyramids, Columns, Wells, Ships, Hills, Clouds, Thunder and *Lightning, Atmosphere, Sun, Moon, Mountains* in the *Moon, Shadows* of *Earth* and *Moon, Beginning* and *End* of *Eclipses*, &c.

In which is also shewn,

A Curious *Trigonometrical Method* of discovering the Places where BEES hive in large WOODS, in order to obtain, more readily, the *salutary Produce* of those *little Insects*.

By the Rev. Mr. TURNER, late of *Magdalen-Hall, Oxford*,
Author of The *View* of the *Earth*;—*View* of the *Heavens*;—*System* of *Gauging*;—and *Chronologer Perpetual*.

Cuncta Trigonus *habet, satagitque docta* Mathesis,
Ille *aperit clausum, quicquid* Olympus *habet*.

Within the grand Triangle *lies unveil'd*,
What Sages *sought for, and what* Heaven *conceal'd*.

LONDON:
Printed for S. CROWDER, in Pater-noster-Row; and S. GAMIDGE, Bookseller, in Worcester. MDCCLXV.

TO THOSE

GENTLEMEN,

Whoſe *Genius* may incline, or *Employment* lead them to the Study of the

MATHEMATICKS.

GENTLEMEN,

TRIGONOMETRY has always been look'd on as one of the moſt uſeful Branches of *Mathematical* Learning. *Navigation*, *Surveying*, *Aſtronomy*, &c. ſtand wholly upon this *Baſis*. But the common Method of anſwering theſe Problems being by large Tables of *Sines*, *Tangents*, and *Secants*, renders it not only expenſive by the purchaſe of them; but often precarious in the Solution, by the Miſtakes of the Preſs. I have therefore, for the Uſe of the *Young Mathematician*, (from a Conſideration of what has been publiſhed on this curious Subject) compoſed the preſent *Syſtem*, by which any of the *Caſes* in *Right* or *Oblique* Plain Triangles may be anſwered on the *Spot*, by an eaſy Calculation in *Arithmetick* only. The great Advantages reſulting from this Method to *Gentlemen* in the *Army* or *Navy*, as well as to thoſe in their *private Studies* at *Home*, muſt immediately appear; as it will be found to anſwer the moſt neceſſary Problems as *expeditiouſly* as *Logarithms*; and at the ſame Time wholly deliver you from thoſe *voluminous Tables* and the *inartificial Fatigues* of carrying them always with you.--- Should this little Treatiſe be ſo happy as to meet your Approbation, it will give a particular Pleaſure to,

Your moſt humble Servant,

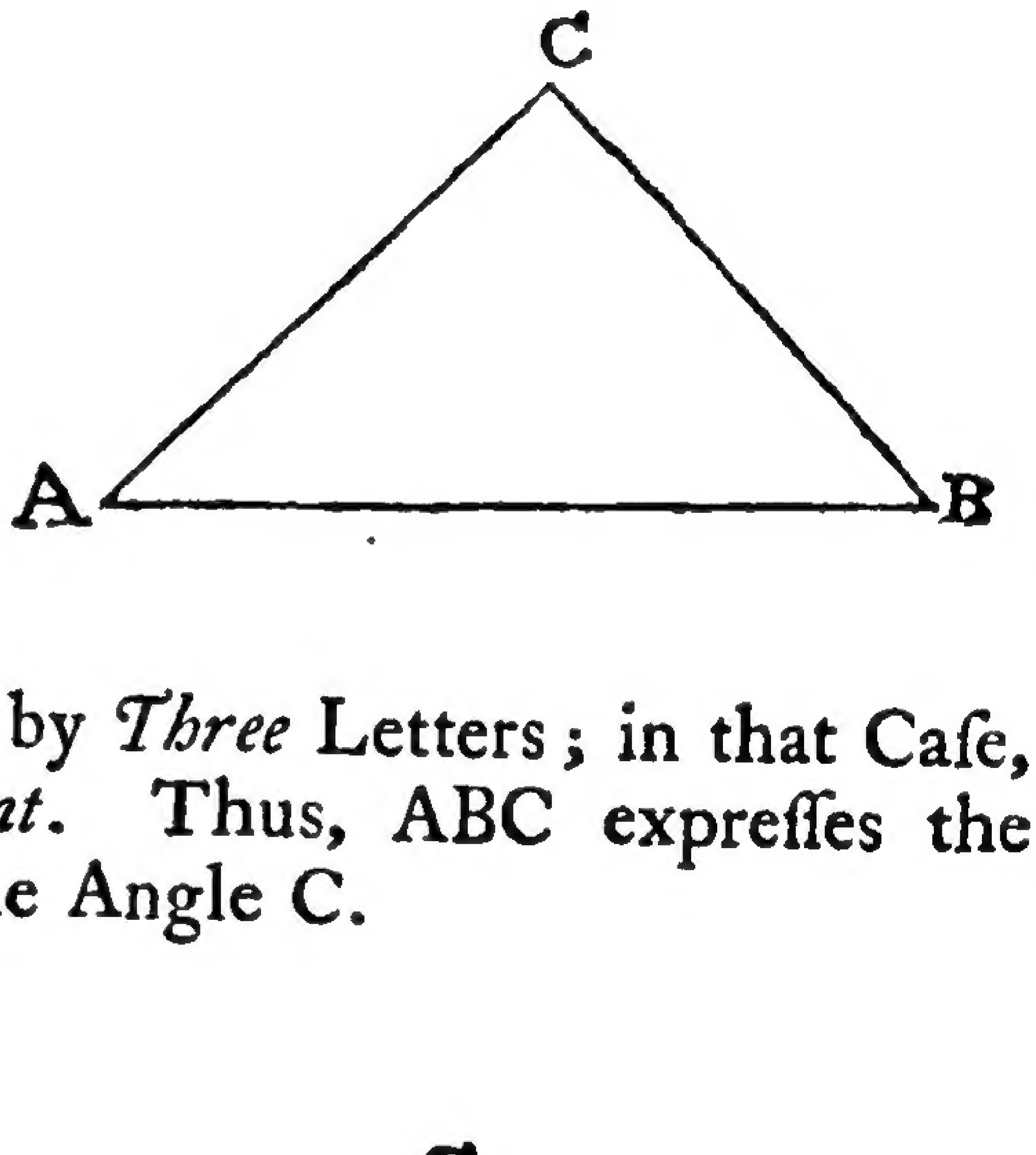
The AUTHOR.

PLAIN

TRIGONOMETRY.

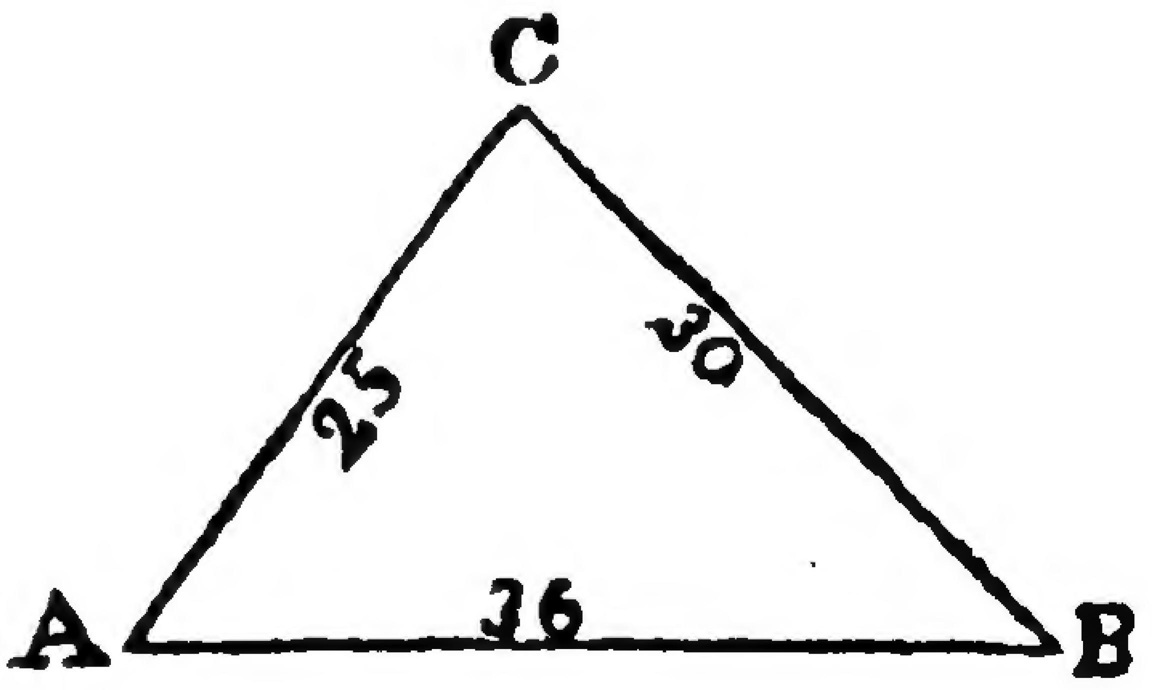
TRIGONOMETRY is that Part of *Mathematicks*, which is employed in calculating the *Sides* and finding the *Angles* of any *Triangle* required; it is of the greatest *Use* in Life, as nothing in *Navigation*, *Astronomy*, &c. can be done without it; and depends on the Knowledge of the following Observations.

(1st.) Every Triangle consists of Six Parts; that is, of *Three Sides* and *Three Angles*, as in the Figure ABC; the Three Sides are, AB, AC, CB, and the Three Angles, A, B, C.

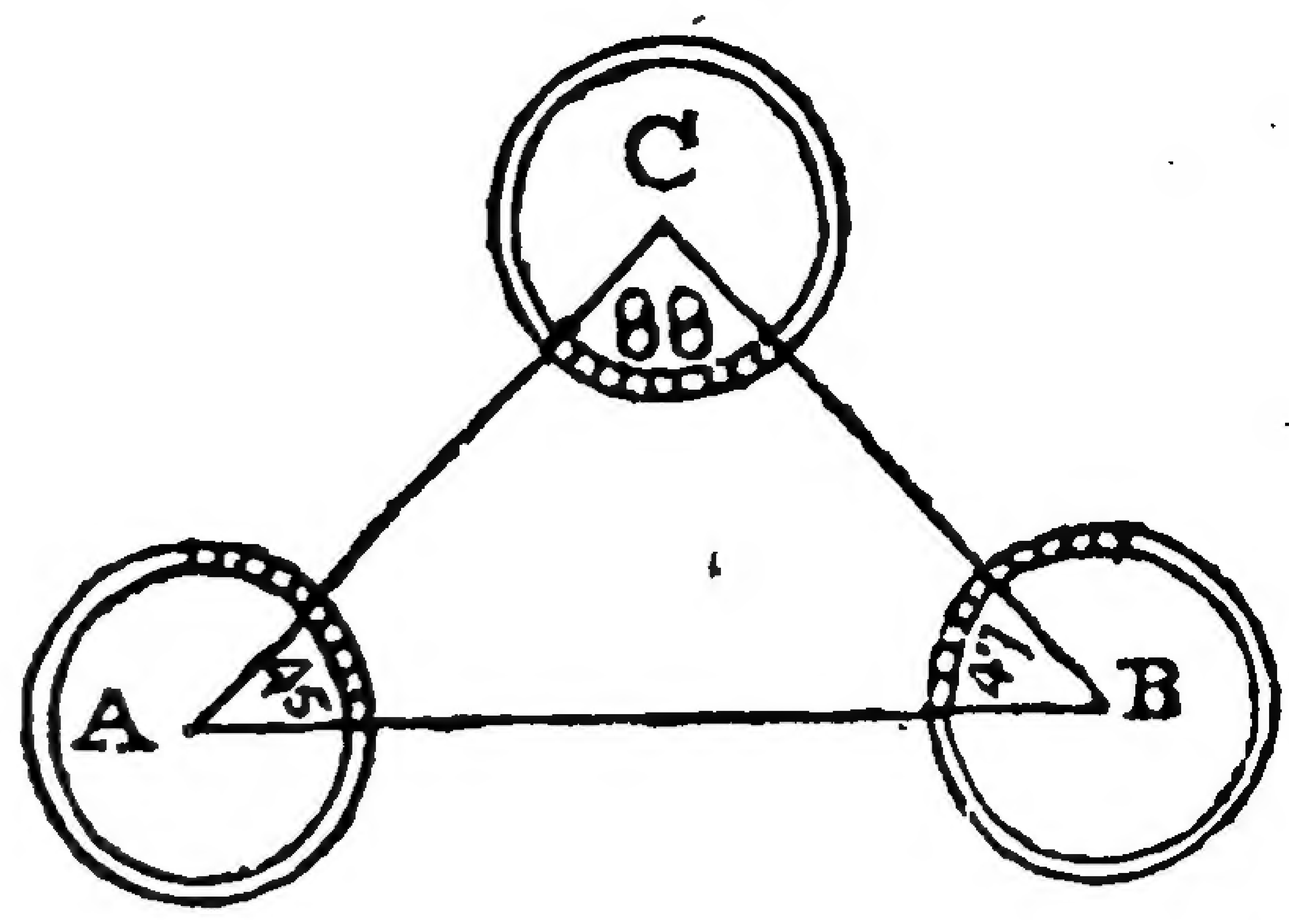


NOTE. Sometimes an *Angle* is expressed by *Three* Letters; in that Case, the *Middle* Letter denotes the *Angular Point*. Thus, ABC expresses the Angle B; BAC the Angle A; and ACB the Angle C.

(2d.) The *Sides* of all plain Triangles are measured by a Line of equal *Parts*, as of *Inches*,—*Feet*,—*Yards*,—or *Leagues*.

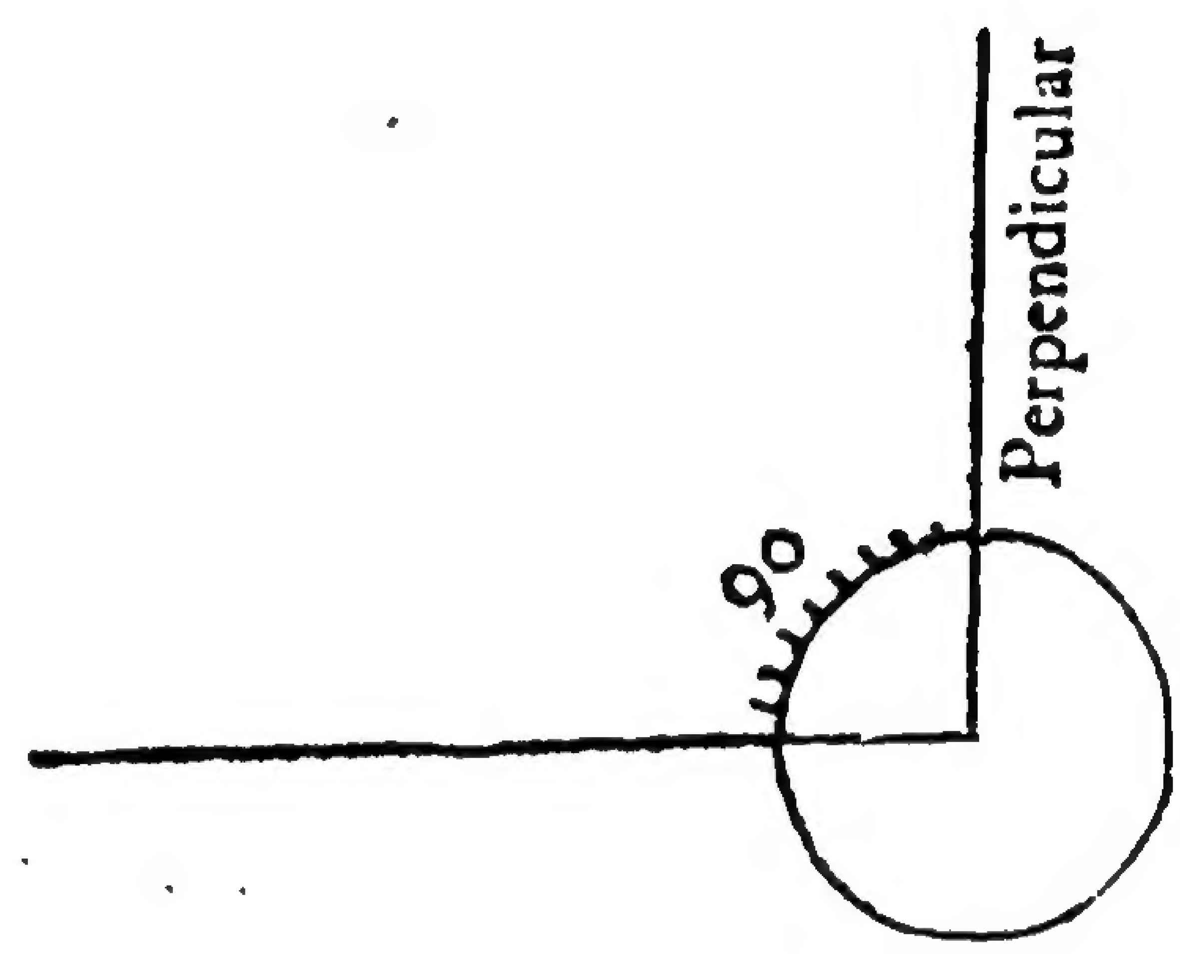


Thus, The Side AB is 36 Leagues.—The Side AC 25 Leagues.—And the Side BC is 30 Leagues.

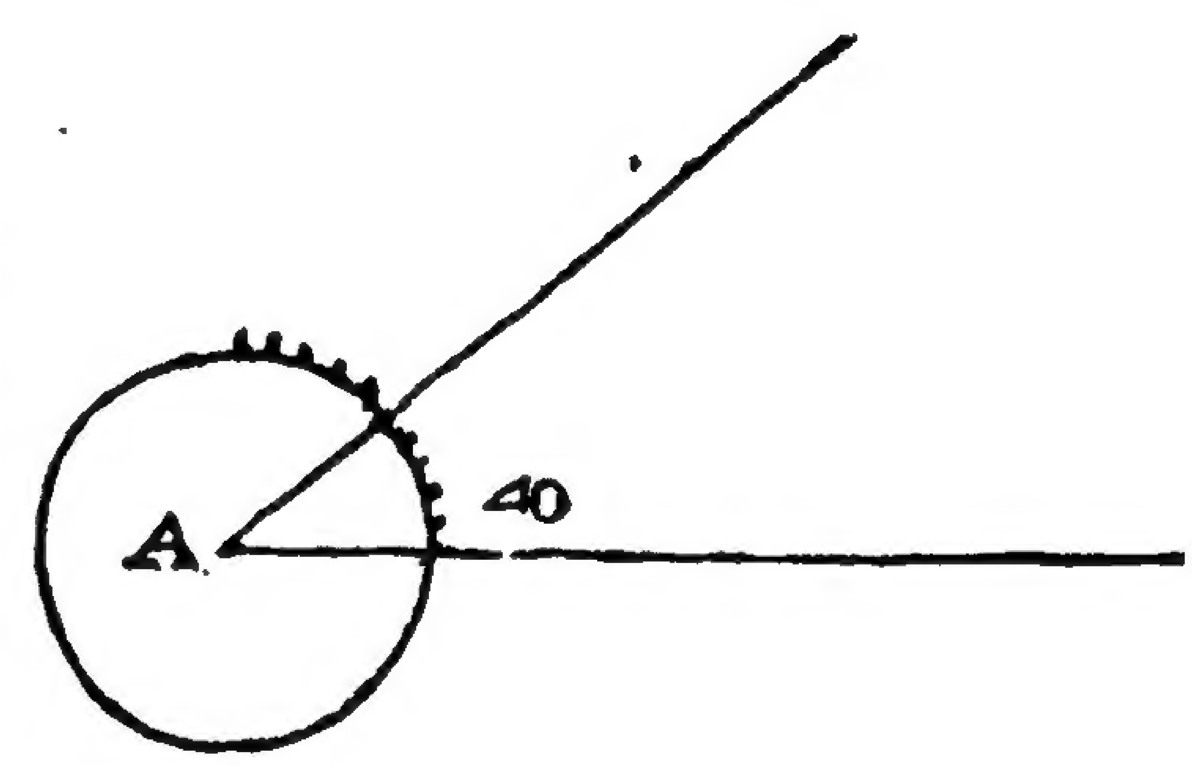


(3d.) The *Angles* are meaſured by the *Arch* of a *Circle* deſcribed upon the Angular Point, and contained between the Two Legs that form the Angle.

NOTE. Every Circle is divided into 360 equal Parts, called *Degrees*; each of which are divided into 60 more, called *Minutes*: And the Number of Degrees contained between the Two Legs, that conſtitute the Angle, is the *Meaſure* of that Angle. Thus, The Angle A is 45 *Degrees.*—The Angle B, 47.—The Angle C, 88.

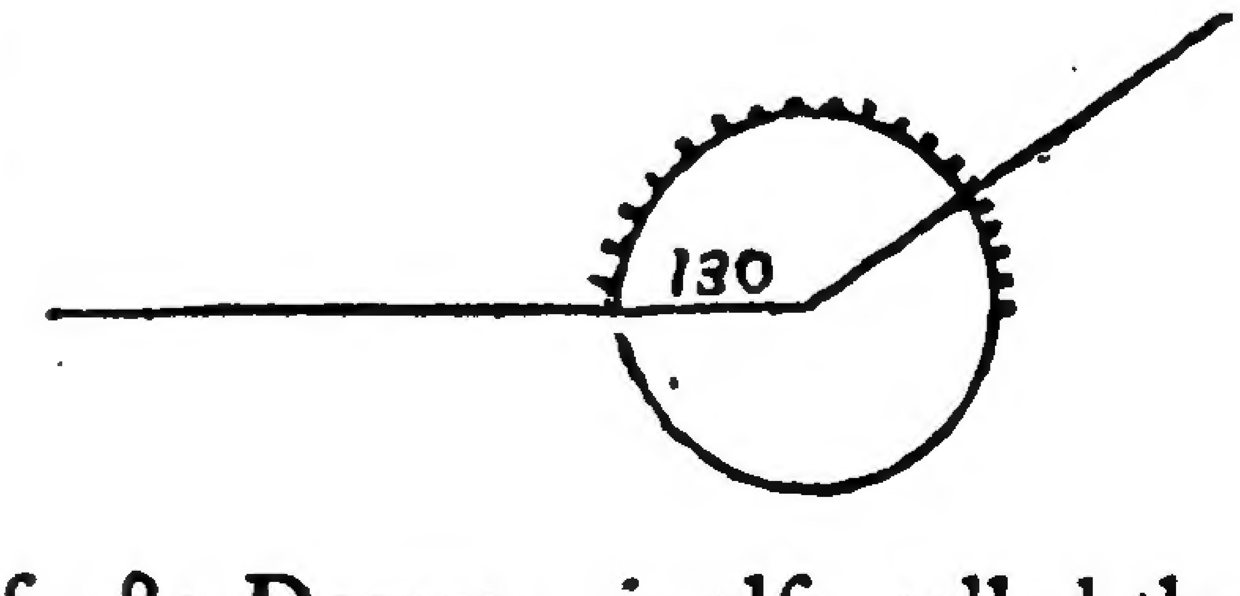


(4th.) If the Arch of a Circle intercepted between the Two Legs be exactly 90 Degrees, the Angle is called a *Right Angle*, and the Legs are *perpendicular* to one another.



(5th.) If the Arch of the Circle be *leſs* than 90 Degrees, the Angle is ſaid to be *Acute*.

NOTE. What an *Acute* Angle wants of 90 Degrees, is called the *Complement* of that Angle. Thus, Suppoſe the Angle A was 40 Degrees; then its Complement is 50 Degrees; for 40 added to 50 make 90, as obſerved before.

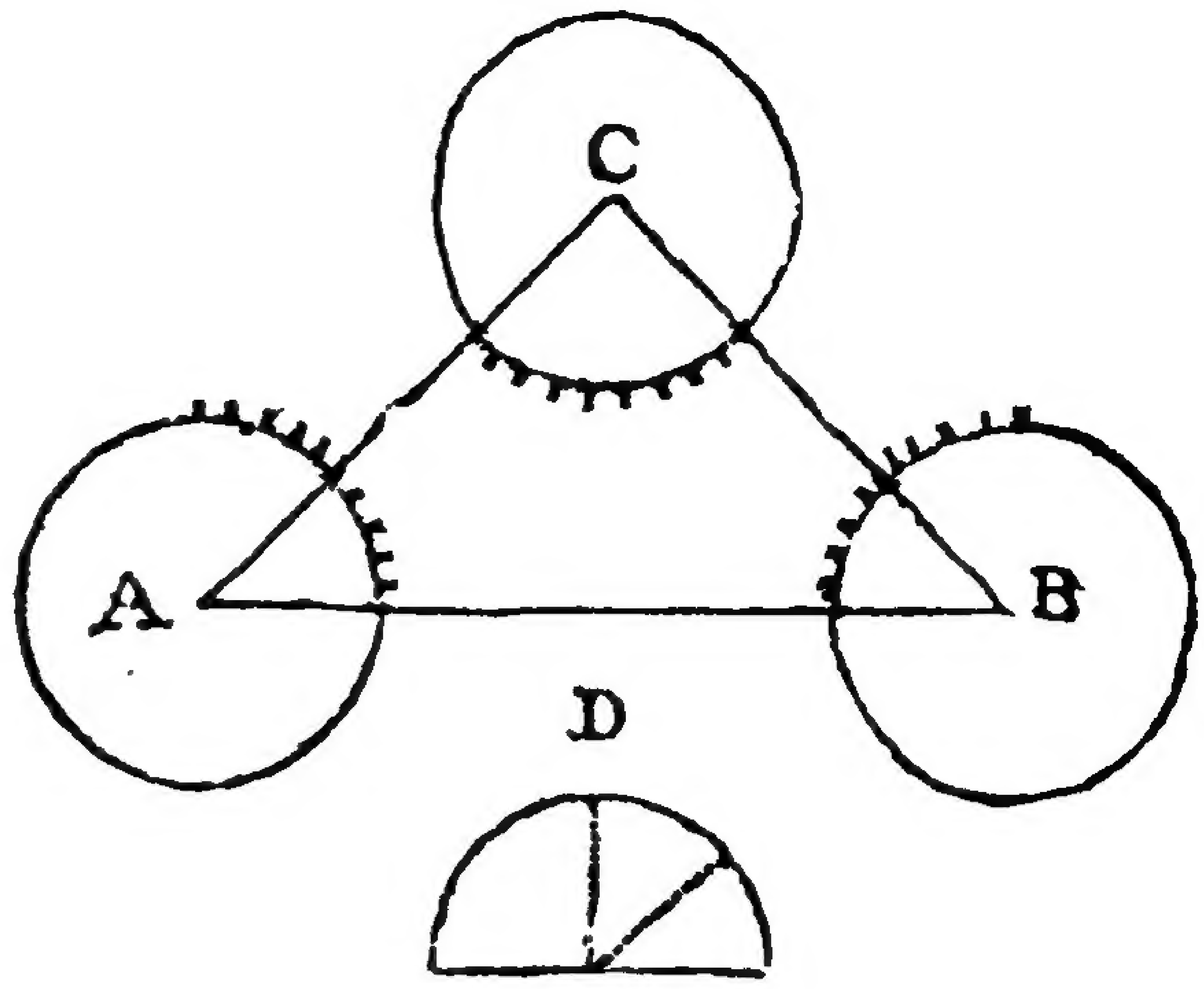


(6th.) If the Arch of a circle be *more* than 90 Degrees, the Angle is ſaid to be *Obtuſe*; and ſo continues to 180 Degrees, where the Angle vaniſhes, the Lines becoming *Strait*.

NOTE. What an *Obtuſe* Angle wants of 180 Degrees, is alſo called the *Complement of that Angle to a Semicircle.*

(7th.) The

(7th.) The Three Angles of every plain Triangle, being taken together, make 180 Degrees (equal to a Semicircle), and this they always do, let the Triangle be drawn however you pleaſe.

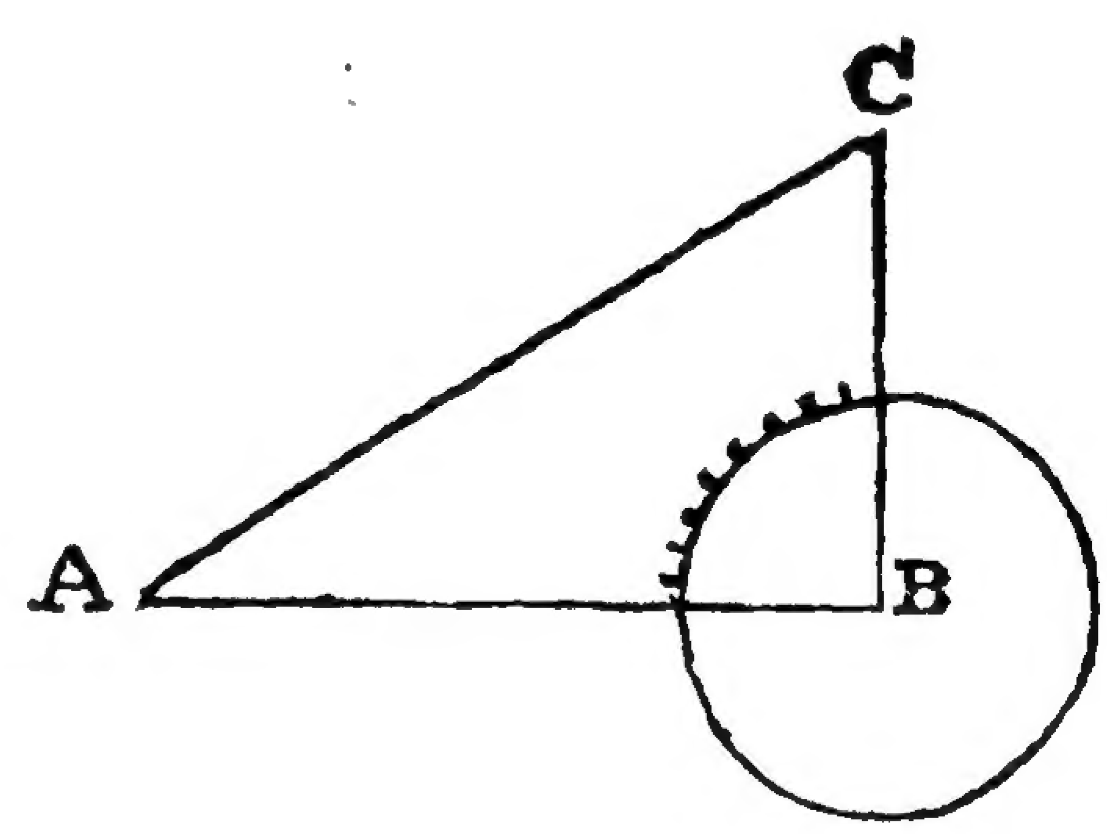


Thus, If the *Semicircle* D be drawn with the ſame *Radius*, or opening of the Dividers, as the little Circles on the Angles A, B, C, are; you will find, by taking off the ſeveral Arches, and applying them to the Semicircle, that they will juſt fill it up, and thereby make 180 Degrees; becauſe every Semicircle contains that Number of Degrees.

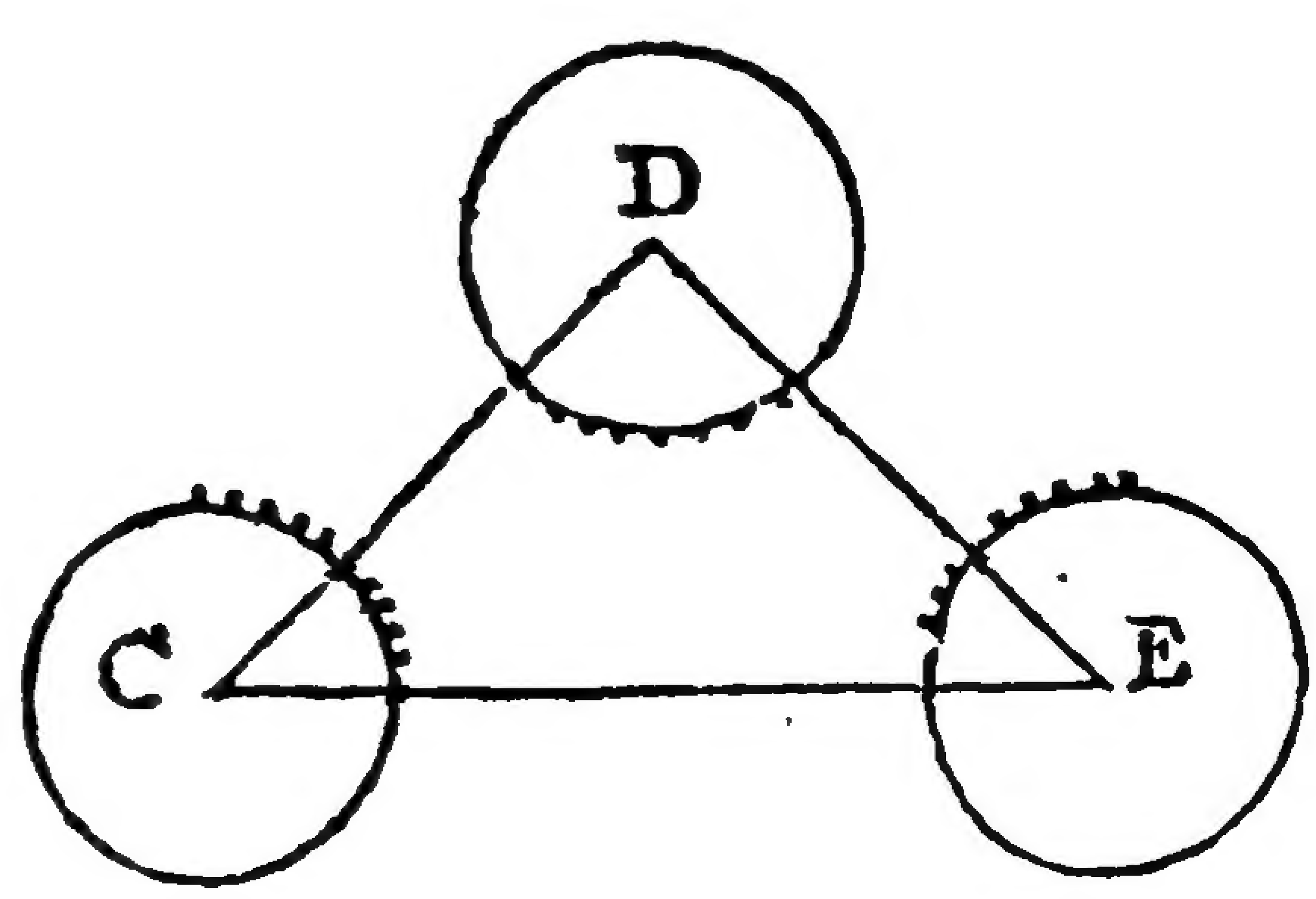
Hence it is evident, that if one Angle be a *Right One*, the other Two will be *Acute*; and taken together, be equal to one *Right Angle*, or juſt 90 Degrees.

Hence alſo, if *Two* Angles of any Triangle are known, the *Third is* eaſily found, being only the *Degrees* the other Two Angles want of 180.

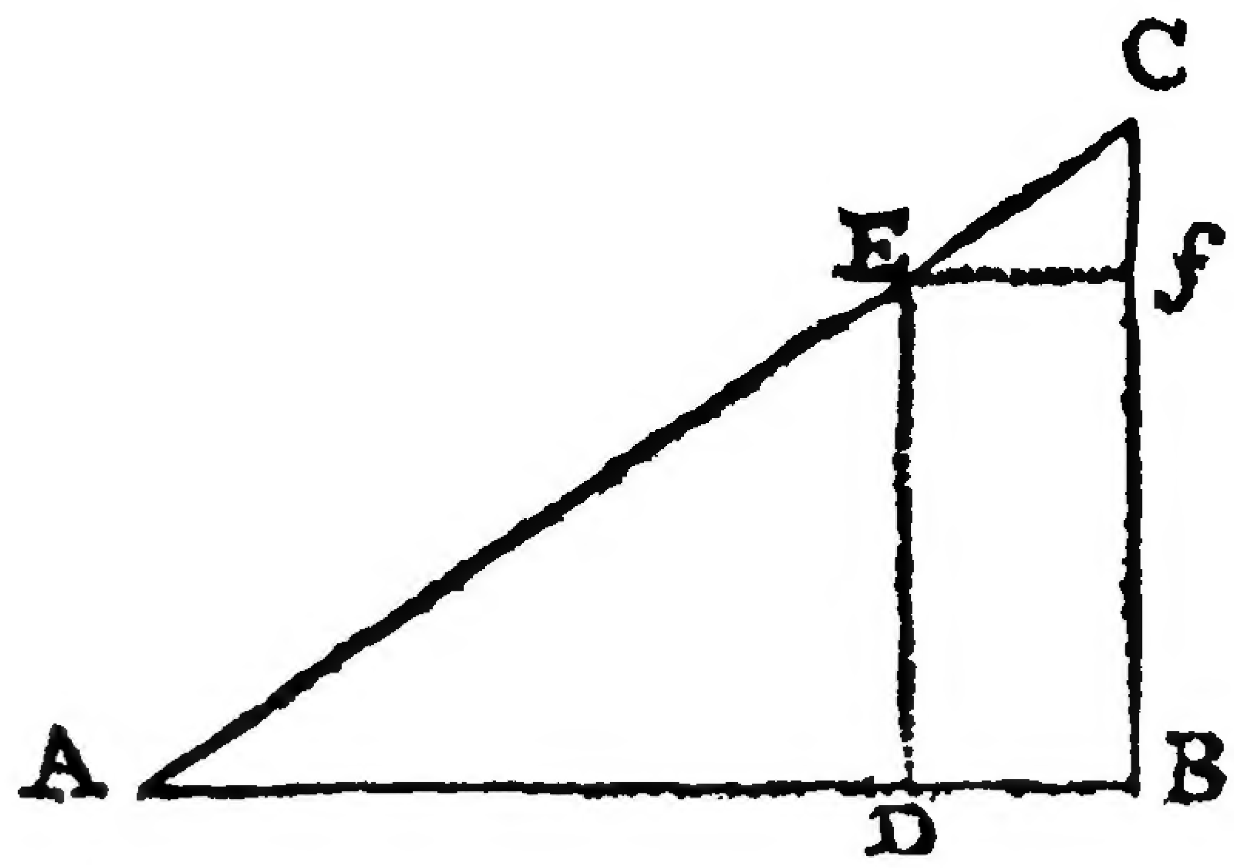
(8th.) If a *Triangle* has one *Right Angle*, it is called a *Right Angled Triangle*: Thus ABC is a Right Angle Triangle, Right-angled at B.—In all Right Angle Triangles, the longeſt Leg is called the *Hypothenuſe*;—the Leg on which it ſtands, the *Baſe*;—and the other Leg, the *Perpendicular*.



(9th.) If neither of the Angles is a Right One, then it is called an *Oblique Angled Triangle*; as the Triangle CDE is an *Oblique Triangle*.



(10th.) If the Angles of one Triangle are equal to the Angles of another Triangle, the Sides of the former are proportional to the Sides of the latter.



Thus, If in the Triangle ABC, you draw the Line ED parallel to CB, the ſmaller Triangle ADE will be *ſimilar to*, i. e. will have the ſame Angles with the larger Triangle ABC.—It will therefore always hold, as

AB : AD : : BC : DE.
Or, as AB : BC : : AD : DE.
Or, as AD : DE : : AB : AC.
Or, as AD : DE : : E*f* : *f*C, *&c.*

(11th.) In all Triangles the *greateſt Side* is oppoſite to the *greateſt Angle*; and on the contrary, the *greateſt Angle* is oppoſite the *greateſt Side.*—If Two *Sides* are *equal*, the oppoſite *Angles* are *equal.*—If all the Sides are equal, then all the Angles are equal to each other.

(12th.) In all Triangles, every Side is in proportion to its oppoſite Angle, and every Angle to its oppoſite Side: And further, as the Angle oppoſite to one Side, is to the Angle oppoſite the other Side, ſo are the Sides themſelves to one another; and the contrary, the Sides to the Angles.

Every *Triangle*, as I obſerved before, conſiſts of Six Parts—*Three Sides* and *Three Angles*. If any Three of the Six Parts (excepting the Three Angles) are given, any one, or every one of the reſt may be found, without the painful Deductions and voluminous Tables of *Logarithms*, *Sines*, *Tangents*, and *Secants*, by the following *Rules* and *Axioms* *.

* This *Method* will be found as exact, as that by the *Logarithms*, if you carry on the *Operation* to *Three* or *Four Decimal* Places; but for common Purpoſes, *One* or *Two Decimal* Places will be near enough. You muſt alſo remember to reduce the *Minutes* of the Angles to *Decimals* of a Degree, which is eaſily done, by allowing One tenth for every Six Minutes.——Or you may turn the Minutes into Decimals thus: As 60, the Minutes in one Degree, : are to the Minutes given, : : ſo are 10, 100, 1000, *&c.* to the *Decimal* required.

Of

Of Right Angled TRIANGLES.

THERE are generally reckoned by Writers on this Subject Seven Cases; but by this Method they are all reduced to Four; the Solutions of which depend on the following *Axioms*.

AXIOM I. Divide 4 Times the Square of the Complement of the Angle, whose opposite Side is either given or sought, by 300 added to 3 Times the said Complement; this Quotient added to the said Angle, will give you an *Artificial Number*, called sometimes the *Natural Radius* *, which will ever bear the same Proportion to the Hypothenuse, as that Angle bears to its opposite Side.——In Angles under 45 Degrees, the *Artificial Number* may be found easier thus: Divide 3 Times the Square of the Angle itself, whose opposite side is given or sought, by 1000; the Quotient added to 57.3 †, a fixed Number, that Sum will be the *Artificial Number* required.—*This is to be used, when the* Angles *and a* Side *are given, to find* another Side.

AXIOM II. The Square of both the Legs, *i. e.* the Square of the *Base* and *Perpendicular* added together, is equal to the Square of the *Hypothenuse*; whose Root is the *Hypothenuse* itself.—*This is made use of, when the* Base *and* Perpendicular *are given, to find the* Hypothenuse.

AXIOM III. The *Sum* of the *Hypothenuse* and *One* of the *Legs* multiplied by their *Difference*, the *Square Root* of that Product will be the *other Leg* required.—*This comes into use, when the* Hypothenuse *and* One Leg *is given, to find the* other Leg.

AXIOM IV. *Half* the *Longer* of the *Two Legs*, added to the *Hypothenuse*, is always in Proportion to 86 ‡, as the *Shorter Leg* is to its *opposite Angle*. —*This is useful, when the* Sides *are given, to find the* Angles.

NOTE. These 4 *Axioms* will answer all the Cases of *Right* and *Oblique Angled Triangles*, except the last Case in Obliques, which will require some further Assistance, and will be shown when we come to treat of that Case.

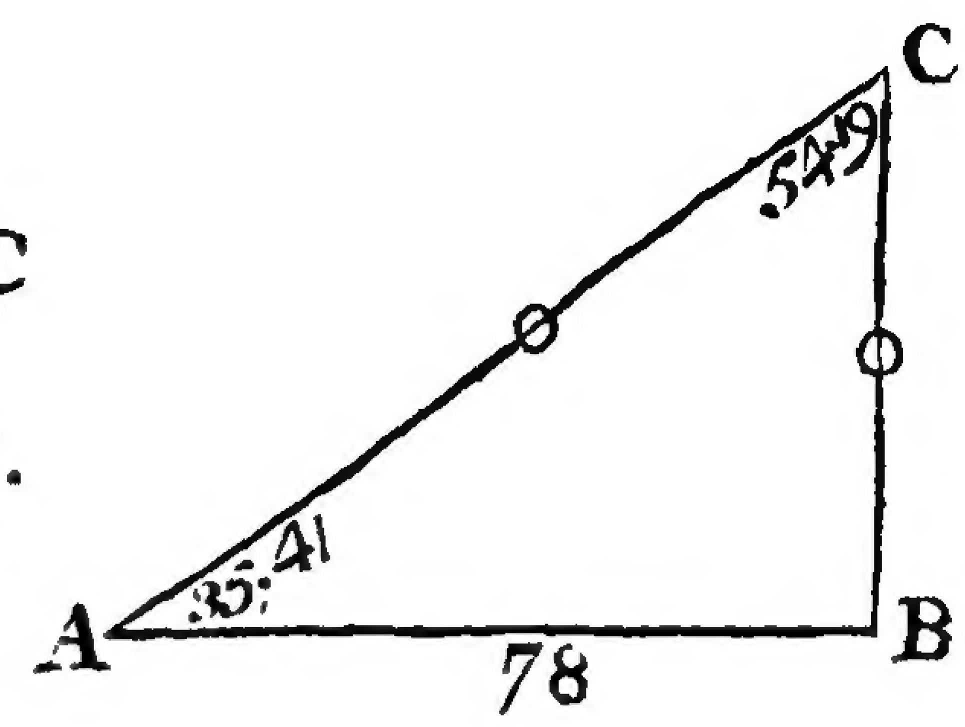
* The *Natural Radius* is only turning the *Right Angle*, = 90 Degrees, into an *artificial Number*, which shall always bear the same Proportion to the *Hypothenuse*, as the given Angle does to its opposite Leg.

† 57.3 is the *Radius* of a Circle whose Circumference is 360.

‡ 86 = *Radius and Half* of a Circle whose Circumference is 360.

CASE I.

The Acute Angles, and one Leg given; to find the Hypothenufe and the other Leg.



Given { Angle A = 35.41, Angle C = 54.19, Bafe AB = 78 } to find { Hypothen. AC and Perpendic. BC.

(1ft.) Find the Natural Radius by Axiom I.

```
      35.7            35.7
      35.7               3
      ----            -----
      2499            107.1
     1785             300.
    1071              -----
    -------           407.1
   1274.49
         4
   -------
407.1)5097.96(12.5
      4071    (54.3
      ----     ----
     10269    66.8 Natural Radius
      8142
     -----
     21276
     20355
     -----
       921
```

(2d.) Find the Hypothenufe by Axiom I.

```
Angle C  :  Bafe  : :  Nat. Rad.
As 54.3 ——— 78 ——————— 66.8
                          78
                        ----
                        5344
                       4676
                       -----
              54.3)5210.4(95.9 + Hypothenufe
                   4887
                   ----
                   3234
                   2715
                   ----
                   5190
                   4887
                   ----
                    303
```

(3d.) Find the Perpendicular by Axiom III.

```
                   96 To Hypothenufe
                   78 Add the Bafe
                   --
                  174 Sum multiply
                   18 by Difference
                  ----
                  1392
                  174
                  ----
Extract the Root  3132(55.9 + Perpendicular
                  25
                  ---
              105)632
                  525
                  ----
            1109)10700
                  9981
                  ----
                   719
```

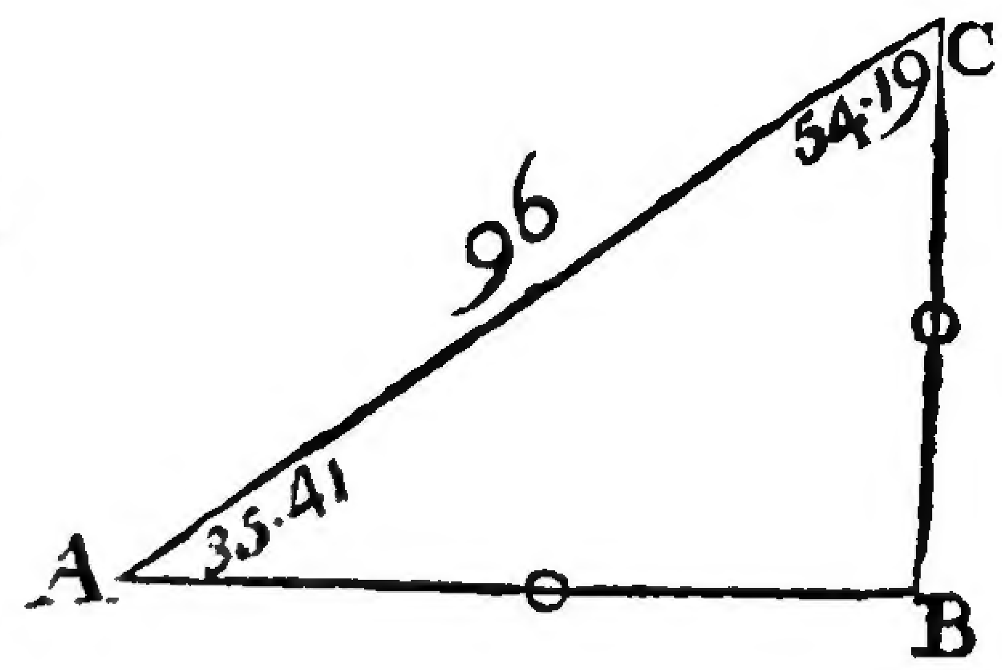
Anfwer, { Hypothenufe, 95.9 +, or 96.
Perpendicular, 55.9 +, or 56. }

CASE

CASE II.

The Hypothenufe and Angles given, to find the Two Legs.

Given { Hypoth. AC = 96, Angle A = 35.41, Angle C = 54.19 } to find { Perpendic. CB and Bafe AB.



(1ft.) Find the Natural Radius by Axiom I.

```
      35.7
      35 7
      ----
      2499
     1785
    1071
    -------
   1274.49
         3
   --------
1.000)3.823.47
   57.3 Add
   ----
   61.1 Natural Radius
```

(2d.) Find the Perpendicular by Axiom I.

```
   Nat. Rad. : Hypoth. :: Angle A
As   61.1    =   96    ——  35.7
                             96
                           ----
                           2142
                          3213
                          ----
                   61.1)3427.2(56 Perpen
                        3055
                        ----
                         3722
                         3666
                         ----
                           56
```

(3d.) Find the Bafe by Axiom III.

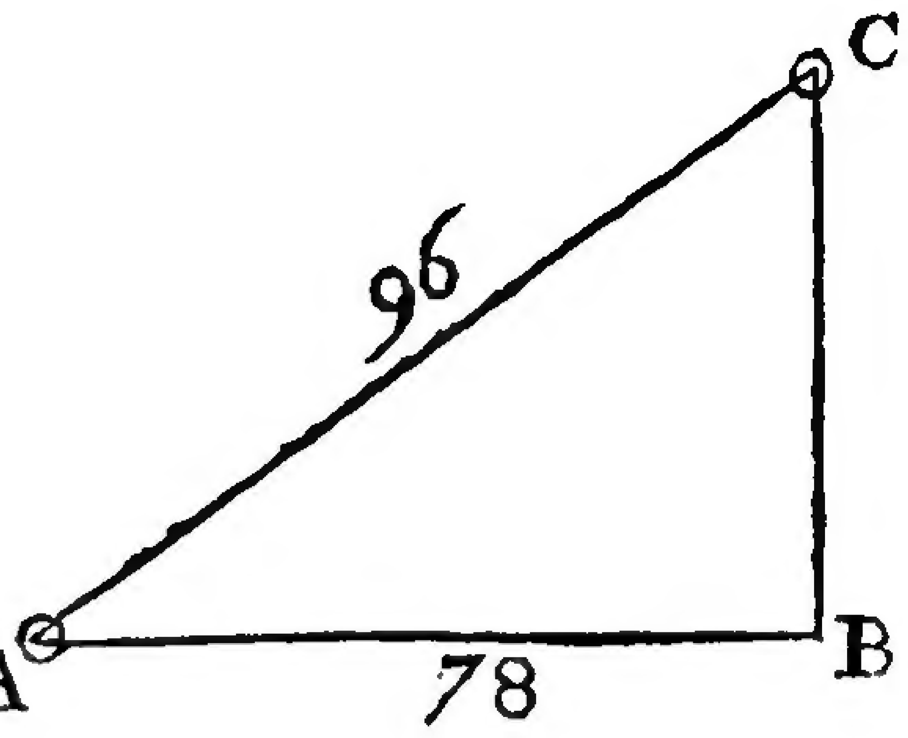
```
                      96 To Hypothenufe
                      56 Add Perpendicular
                     ---
                     152 Sum multiplied by
                      40 The Difference
                    ----
Extract the Root   6080(77.9 + Bafe
                   49
                  ----
             147)1180
                 1029
                 ----
           1547)15100
                13941
                -----
                 1169
```

Anfwer, { Perpendicular, 56. Bafe, 77.9 +, or 78.

CASE

CASE III.

The Hypothenuſe and One Leg given, to find the Angles and the other Leg.



Given { Baſe AB = 78, Hypoth. AC = 96 } to find { Perpendic. CB and Angles A & C.

(1ſt.) Find the Perpendicular by Axiom III.

```
                     96  To Hypothenuſe
                     78  Add the Baſe
                    ---
                    174  Sum multiply
                     18  By Difference
                   ----
                   1392
                   174
                   ----
Extract the Root   3132(55,9 + Perpendicular
                   25
                   ---
               105)632
                   525
                   -----
              1109)10700
                    9981
                    ----
                     719
```

(2d.) Find the Angle by Axiom IV.

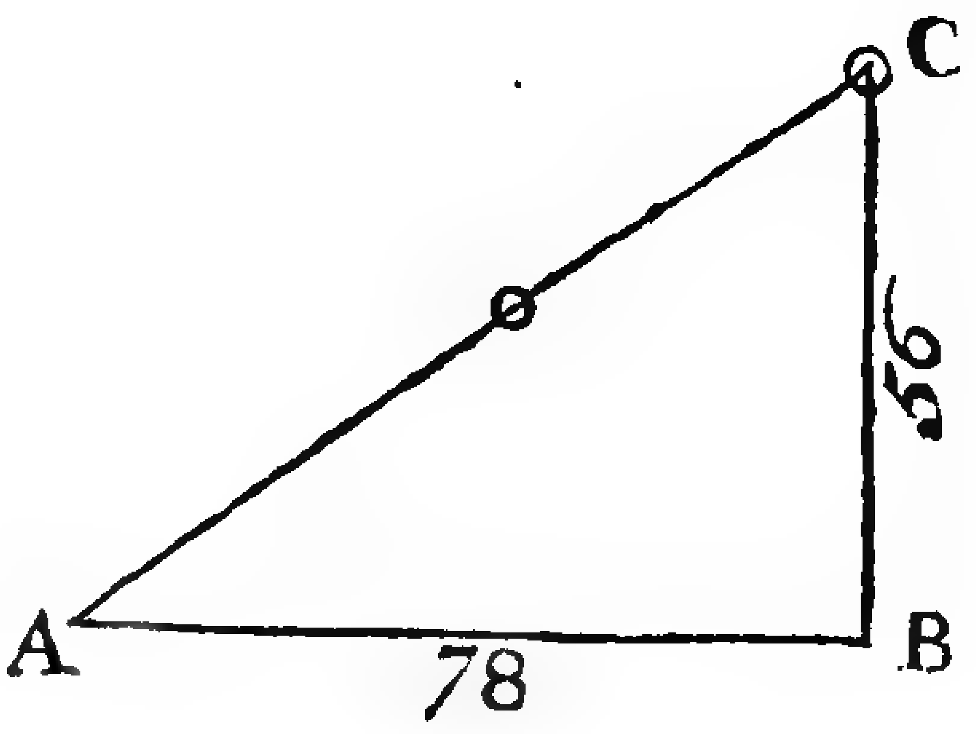
```
To  Hypothenuſe  96
Add half longer Leg 39
                 ---  :  Fixed Number  : :  Perpendicular
Sum             135  ------ 86 ------        56
                                             86
                                            ---
                                            336
                                           448
                                           ----
                                     135)4816.(35.67 + Angle A
                                         405
                                         ---
                                          766
                                          675
                                          ---
                                           910
                                           810
                                           ----
                                           1000
                                            945
                                            ---
                                             55
```

Anſwer, { The Angle, 35° 41′ nearly.
The Perpendicular, 55.9 +, or 56.

CASE IV.

The Two Legs given; to find the Hypothenuſe and the Angles.

Given { Baſe AB = 78, Perpend. CB = 56 } to find { Hypoth. AC and Angles A & C.



(1ſt.) Find the Hypothenuſe by Axiom II.

```
Perpendicular 56        78  Baſe
    ſquar'd   56        78  ſquar'd
             ---        ---
             336        624
            280        546
            ----       ----
            3136       6084 Square of Baſe
            ----       3136 Square of Perpendic.
                       ----
      Extract the Root 9220(96 The Hypothenuſe
                       81
                       ---
                   186)1120
                       1116
                       ----
                          4

To  Hypothenuſe  96
Add half longer Leg 39
                 ---  :  Fixed Number  : :  Perpendicular
           Sum  135 ---- 86 ----              56
                                              86
                                             ---
                                             336
                                            448
                                            ----
                                      135)4816.(35.674+ Angle A
                                            405
                                            ---
                                             766
                                             675
                                             ---
                                              910
                                              810
                                              ---
                                              1000
                                               945
                                              ----
                                                550
                                                540
                                                ---
                                                 10
```

Anſwer, { The Hypothenuſe, 96. The Angle, 35°.674, 35° 41′ nearly.

 NOTE.

NOTE. Thus all the Cafes of *Right Angled Triangles*, are eafily and readily anfwered: and by the fame Rules, and with the like Eafe may the *Oblique Angled Triangles* be anfwered, as will evidently appear in the following Cafes.

Of Oblique TRIANGLES.

IN the Solution of an *Oblique Triangle*, it is neceffary, by this Method, to divide it into Two *Right Angled Triangles*, by means of a *Perpendicular*, which muft always fall upon the End of a given Side, and oppofite to a given Angle.

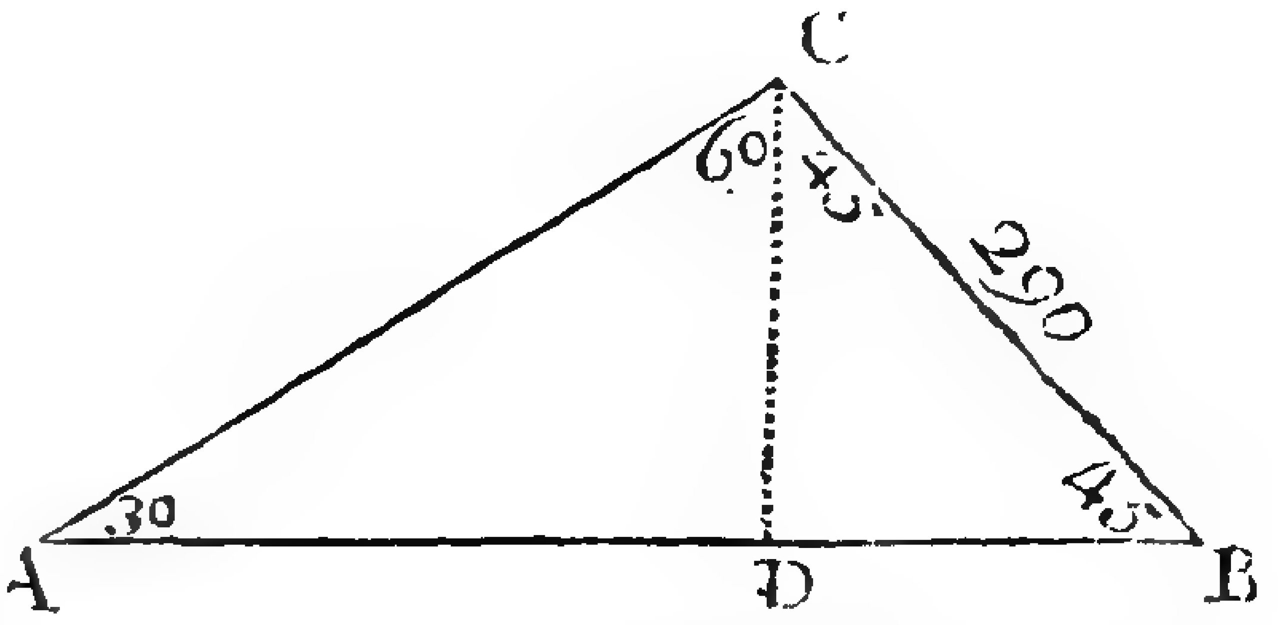
By this means the *Perpendicular* will fometimes fall *within*, and fometimes *without* the Triangle: When it falls within, it falls upon fome Part of the Bafe, or longeft Side; but when it falls without, it falls upon one of the fhorter Sides continued. In either Cafe, there are Two *Right Angled Triangles* made, and the Angles, or Sides fought, are found as if they were Parts of *a Right Angle*, by the foregoing Axioms; but it requires *Two* or *Three* Operations.

CASE

CASE I.

Two Angles, and a Side opposite to one of them, given; to find the other two Sides.

Given { Angle A 30, Angle B 45, Side BC 290 } to find { Side AC and Side AB.




```
 45        45
 45         3
---       ---
225       135
180       300
---       ---
2025      435
   4
----
435)8100(18.6
    434  45.
    ---  ---
    3750 63.6 Natural Radius
    3480
    ----
     2700
     2610
     ----
```

```
  30
  30
 ---
 900
   3
 ---
2.700
57.3
----
Natural Radius 60.0
```

(1st.) Find the Perpendicular CD.

```
   N. Rad.     Hypoth.    ∠ at B
As 63.6 ———— 290 ———— 45
                45
              ----
              1450
             1160
             ----
63.6)13050(205 Perpendicular CD
     1272
     ----
      3300
      3180
      ----
       120
```

(2d.) Find the Side AC.

```
 ∠ at A    Perp. CD   N. Rad.
As 30 ——— 205 ——— 60
           60
          ---
30)12300(410 The Side AC Hypothenuse
   120
   ---
    30
    30
    --
     0
```

(3d.) Find the Side AD.

```
To Hypoth. AC   410
Add Perpend. CD 205
                ---
Sum             615
Multiply by Difference 205
                ----
               3075
              12300
              -----
Extract the Root 126075(354.9 AD
                 9
                 --
             65)360
                325
                ---
            704)3575
                2819
                ----
           7089)75900
                63801
                -----
                12099
```

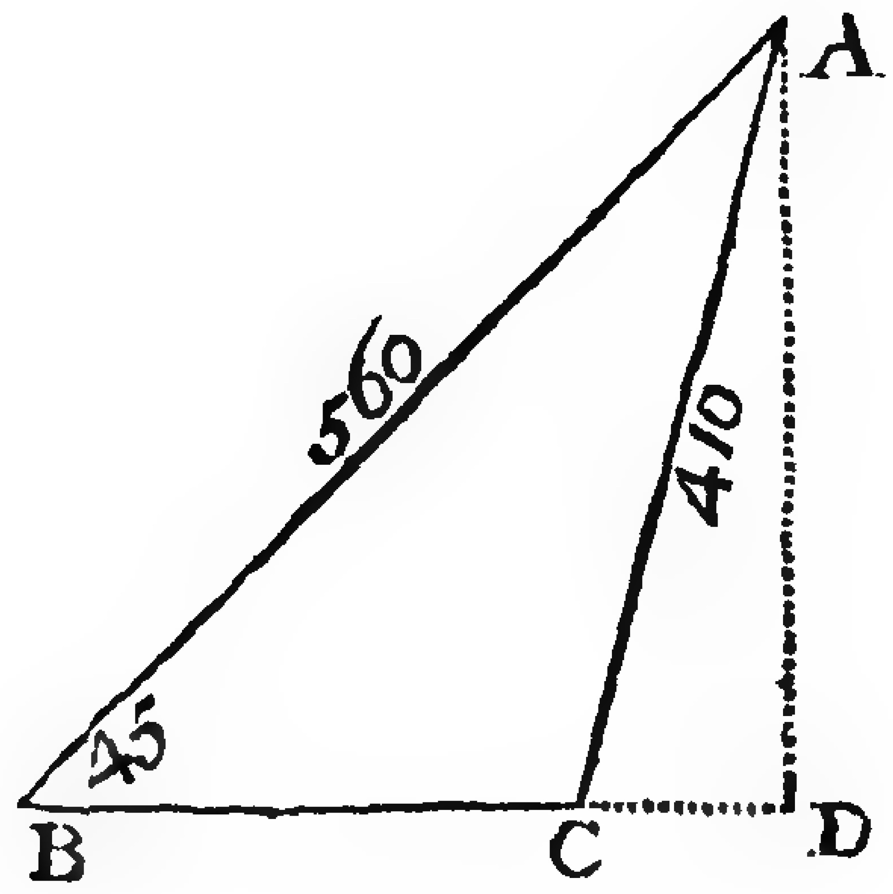
(4th.) Find the Side DB.

```
To Hypoth CB    290
Add Perpend. CD 205
                ---
Sum             495
Multiply by Difference 85
                ----
               2475
              3960
              -----
Extract the Root 42075(205 DB
                 4
                 --
             405)2075
                 2025
                 ----
                   50
```

Answer, { The Side AC 410. The Side AB 560. The Angle C 105.

CASE II.

Two Sides, and an Angle oppofite to One of them, being given; to find the reft.



Given		to find	
	Side AB 560		Angle C
	Side AC 410		and
	Angle B 45°		Side BC.

(3d.) Find the Angle DAC.

```
     To Hyp. CA 410
Add ½ Perp. AD 198
               ———   Fix'd Num.    Side CD
          As 608 ——— 86 ———————— 106
                                  86
                                  ——
                                  636
                                 848
                                 ———
                         608)9116(14.9+, or 15° ∠ A
                             608
                             ———
                             3036
                             2432
                             ————
                             6040
                             5472
                             ————
                              568
```

(4th.) Find Side BD.

```
  To Hypoth BA 560
Add Perpend. DA 396
                ———
            Sum 956
Multiply by Differ. 164
                ———
               3824
              5736
              956
              —————
Extract the Root 156784(395.9 + or 396 BD
                 9..
                 ——
             69)667
                621
                ———
            785)4685
                3925
                ————
           7909)75900
                71181
                —————
                 4719
```

Then from BD = 396
Take —— CD = 106
———
Remains — BC = 290 Required.

(1ft.) Find the Side AD.

```
     Nat. Rad.      Hyp. BA      ∠ B
As    63.6 ———————   560 ———————  45
                      45
                     ————
                     2800
                     2240
                     —————
               93.6)25200.0(396 AD
                     1908
                     ————
                      6120
                      5724
                      ————
                       3960
                       3816
                       ————
                        144
```

(2d.) Find CD.

```
   To Hypoth. CA 410
Add Perpend. AD 396
                ———
            Sum 806
Multiply by Difference 14
                ———
               3224
               806
               ————
Extract the Root 11284(106 CD
                 1
                 ———
             206)1284
                 1236
                 ————
                   48
```

Hence we find by Infpection,
The Angle ACD = 75
The Angle ACB = 105
and The Angle BAC = 30° Required.

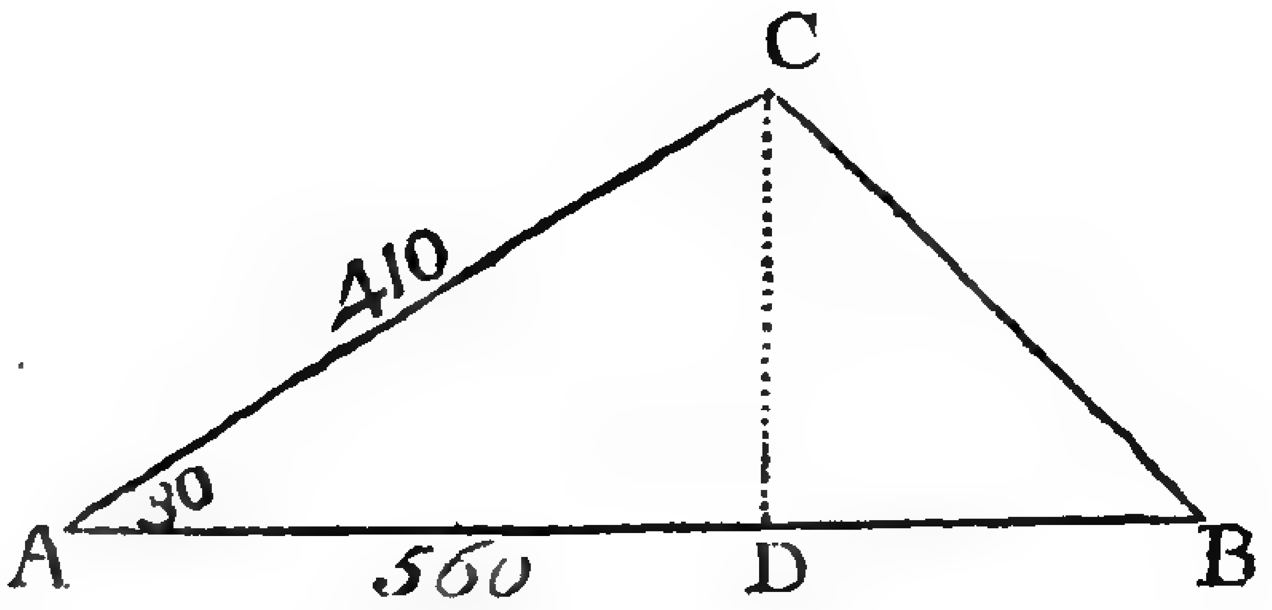
*** In this, and feveral of the following *Cafes* and *Problems*, the Operation for finding the *Natural Radius* is omitted, for Want of Room.

CASE

CASE III.

Two Sides, with the Angle comprehended by them, given; to find the rest.

Given { Side AC 410, Side AB 560, Angle A 30° } to find { Side BC and Angles B & C.



(1st.) Find the Perpendicular CD.

```
     N. Rad.    Hypoth. AC   ∠ at A
As  60 ———— 410 ———— 30
                30
           ————
        60)12300(205 Perpendicular CD
           120
           ————
            300
            300
            ———
              0
```

(2d.) Find the Part AD.

```
  To Hypoth. AC 410
 Add Perpend. CD 205
                ————
            Sum 615
Multiply by Differ. 205
                ————
                3075
               12300
               ——————
Extract the Root 126075(355 Base AD
                 9....
                ———
               65)360
                  325
                 ————
              705)3575
                  3525
                  ————
                    50

   From AB = 560
   Take AD = 355
             ———
Remains BD = 205
```

(3d.) Find the Side BC.

```
BD squar'd 205          205 CD squar'd
           205          205
           ————         ————
           1025         1025
           4100         4100
           —————        —————
To Square of DB 42025   42025
Add Square of CD 42025
                 —————
Extract the Root 84050(289.9 +, or 290 BC
                 4
                 ——
              48)440
                 384
                 ————
             569)5650
                 5121
                 —————
            5789)52900
                 52101
                 —————
                   799
```

(4th.) Find the Angle B.

```
 To 290   the Side BC
Add 102 5 half DB
   ————      Fix'd Num.   Side DC
As 392.5 ———— 86 ———— 205
                       86
                      ————
                      1230
                      1640
                      —————
            3925)17630.0(44.9 +, or 45° ∠B
                 15700
                 —————
                  19300
                  15700
                  —————
                  36000
                  35325
                  —————
                    675
```

The Angle A = 30, added to Angle B = 45, and then subtracted from 180, leaves 105 for Angle C.

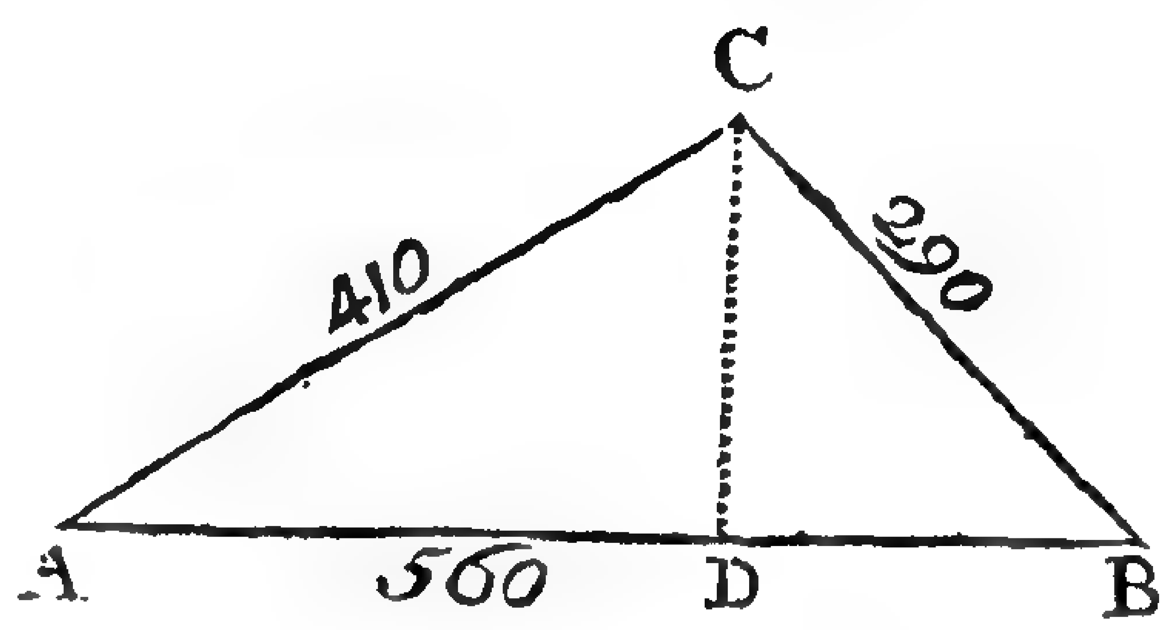
Answer, { The Side BC 290, The Angle B 45°, The Angle C 105°.

CASE IV.

The Three Sides given; to find the three Angles.

In all Triangles, as the Base or greater Side is to the Sum of the other Two Sides; so is the Difference of the Sides to the Difference of the Segments of the Base; which Half Difference, added to Half the Base, the Sum will be the *Greater Segment*, upon which the Perpendicular falls: But if subtracted from half the Base, the Remainder will be the *Less Segment*: So will the *Oblique Triangle* be reduced to two Right Angled ones, and may be answered after the same Manner as before.

Given { Side AB 560, Side AC 410, Side BC 290 } to find { Angle A, Angle B, Angle C. }



(1st.) Find the Segments of the Base.

```
As 560 ——— 700 ——— 120
           120
           -----
           14000
           770
           -----
       560)84000(150 Difference
           560
           -----
           2800
           2800
           -----
              0
```

To ½ Base 280
Add ½ Diff. 75
Gr. Seg. AD 355

From ½ Base 280
Subtract ½ Diff. 75
Less Seg. DB 205

(2d.) Find the Perpendicular CD.

```
To Hypoth. CA  410
Add Gr. Seg. AD 355
               ---
Sum            765
Multiply by Difference 55
               ----
               3825
              3825
              -----
Extract the Root 42075(205 CD
                 4
                 ----
             405)2075
                 2025
                 ----
                   50
```

(3d.) Find the Angle at A.

```
To  410 Side AC
Add 177 half AD
    ---     Fix'd Num.    Side DC
As  587 ——— 86 ———        205
                           86
                          ----
                          1230
                         1640
                         -----
                   587)17630(30 ∠ A
                       1761
                       ----
                         20
```

(4th.) Find the Angle at B.

```
To  290 Side BC
Add 102 half DB
    ---     Fix'd Num.   Side DC
As  392 ——— 86 ———       205
                          86
                         ----
                         1230
                        1640
                        -----
              392)17630(44.9 +, or 45° ∠ B
                  1568
                  ----
                  1950
                  1568
                  ----
                   3820
                   3528
                   ----
                    292
```

Then the Angle A = 30°, added to B = 45°, and subtracted from 180°, leaves 105° for the Angle C, which were the Angles required.

The

The Use of TRIGONOMETRY

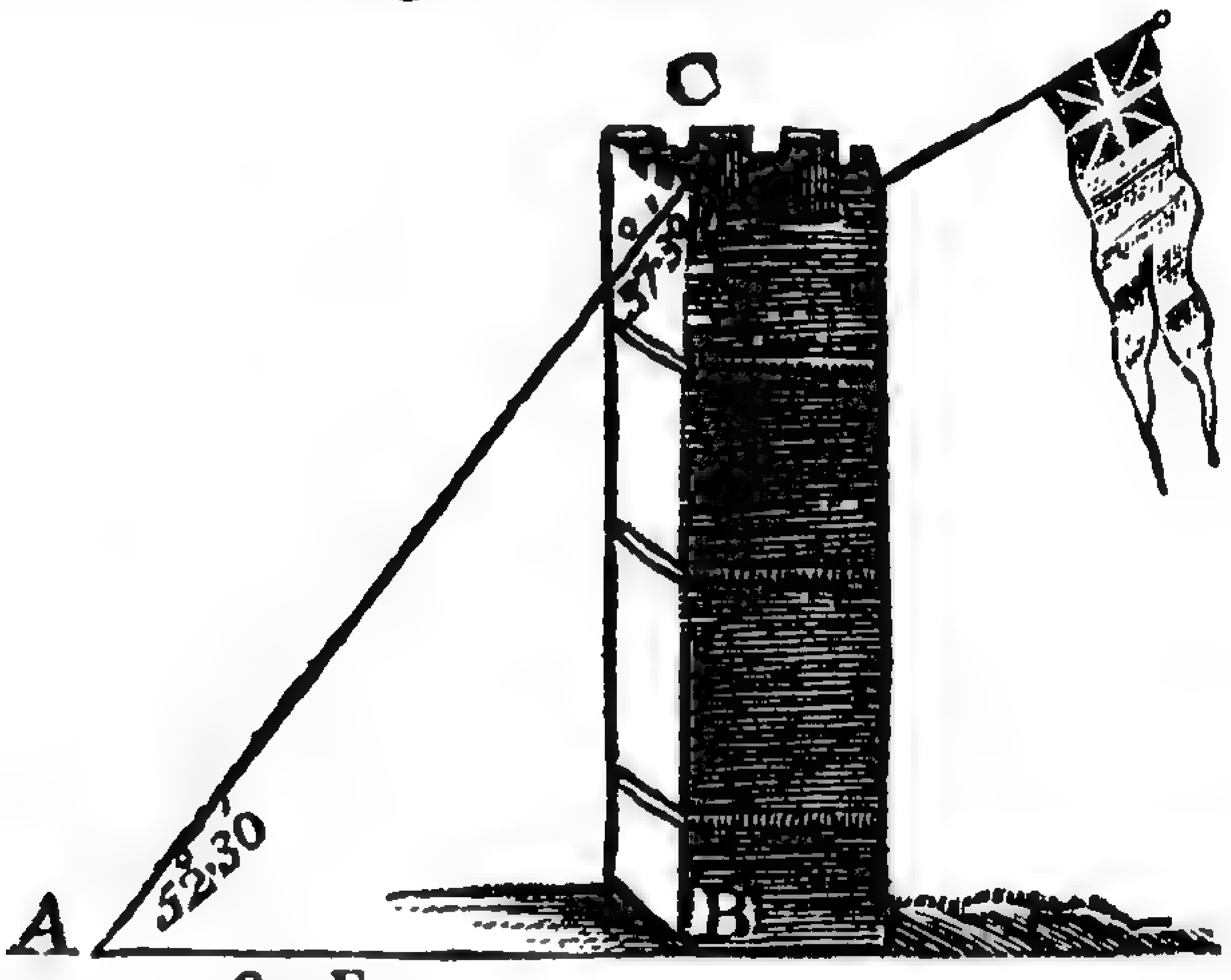
Exhibited in the Solutions *of a Number of interesting* Problems; *many of which every Day occur; are of the greatest Utility in the* Army, Navy, *&c. and cannot be answer'd without it.*

PROBLEM I.

To take the Height of any *accessible Object* at one Station.

First, with a *Quadrant*, by looking through the Sights to the Top of the Tower, find the Quantity of the Angle A, which suppose 52° 30′; then measure the Distance AB, which suppose to be 85 Feet; from hence you may proceed to find the *Height*, by Case I. of Right Angled Triangles.

```
      37 5
      37 5
      ----
      1875
     2625
    1125
    -------
   1406.25
         3
   -------
1000)4.218.75
     57.3
   --------
   61.51875 Natural Radius
   --------
But 61.52 is exact enough in Practice.
```



85 Feet

(1st.) Find the Hypothenuse AC.

```
   Angle C  :  Base  ::  Nat. Rad.
As 37.5  ———  85  ———   61.52
                           85
                        -----
                        30760
                       49216
                       -----
             37.5)5229.20.(139.44 + Hyp.
                  375
                  ---
                  1479
                  1125
                  ----
                   3542
                   3375
                   ----
                    1670
                    1500
                    ----
                     1700
                     1500
                     ----
                      200
```

(2d.) Find the Perpendicular BC.

```
       To Hypothenuse  139.44
             Add Base   85.
                       ------
                  Sum  224.44
Multiply by Difference  54.44
                       ------
                        89776
                       89776
                      89776
                     112220
                    -----------
Extract the Root   12218.5136(110.537 Perpendic.
                   1........
                   --
                 21)22
                    21
                    ----
               2205)11851
                    11025
                    -----
              22103)82636
                    66309
                    ------
             221067)1632700
                    1547469
                    -------
                      85231
```

Answer, 110.537 +, or 110 Feet, and above ½ : The Height Required.

Note; That in this, and all such Cases, you must add the *Height* of your *Eye*, or *Instrument* to the *Altitude* before found.

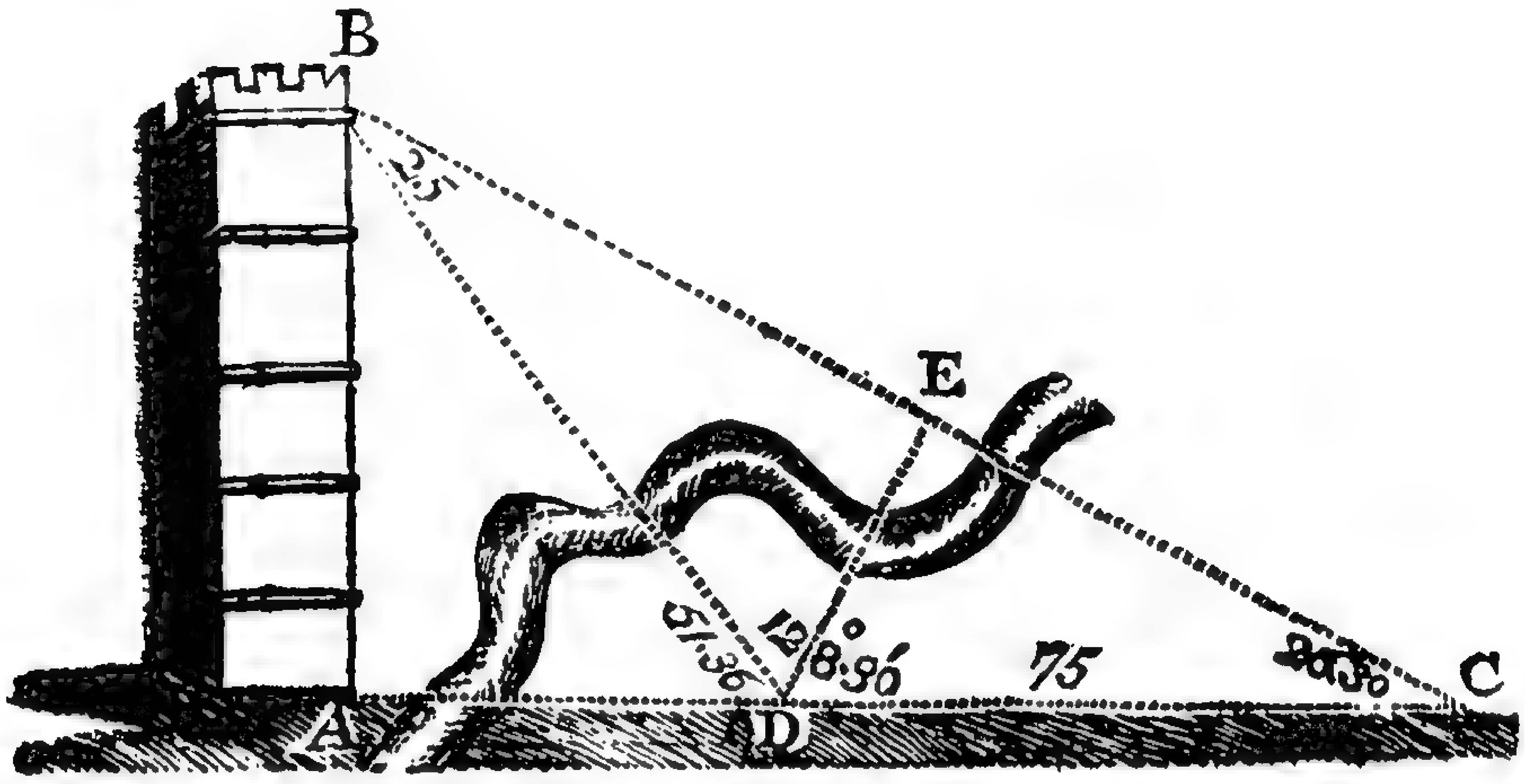
PROBLEM

PROBLEM II.

To meaſure an *Inacceſſible Altitude.*

Let AB, in the following Figure, be a *Church, Tower,* or *Fort,* whoſe Height is required; but by Reaſon of a River, or ſome other *Obſtacle,* it is *inacceſſible*; that is, you cannot come to the Foot of it, by Reaſon of the *Water,* &c.

Firſt, with a *Quadrant,* take the Angle of *Altitude* at C, which ſuppoſe 26° 30′. Then meaſure in a Right Line towards the *Tower* to D, any Diſtance, ſuppoſe 75 Feet, and at D obſerve again the Angle of *Altitude,* which let be 51° 30′.



Then; the two Viſual Lines CB and DB, with the Diſtance DC, make the Oblique Triangle CBD, in which are given—All the Angles and Side CD. The Angles BCD being 26° 30′, and the Complement of ADB 51° 30′ to 180, is the Obtuſe Angle BDC 128° 30′. Conſequently, the third Angle CBD, at the Top, is = 25°.

(1ſt.) Find the Perpendicular DE in Triangle DBC.

Nat. Rad. :	Op. Side DC ::	Ang. C :	Perp. DE
As 59.4 —	75 ——	26.5 —	33.46

(2d.) Find the Viſual Line BD in Triangle BDE.

Ang. B :	Op. Side DE ::	N. Rad. :	Side BD
As 25 ——	33.46 ——	59.17 —	79.19

(3d.) Find the Height AB in the Right Angled Triangle ABD.

Nat. Rad. :	Op. Side BD ::	Ang. D :	Height
As 65.7 ——	79.19 —	51.5 —	62 AB

(4th.) Find the Diſtance AD in the Triangle ABD.

N. Rad :	Op. Side BD ::	Ang. ABD :	Diſ. AD
As 61.7 —	79 19 ——	38.5 —	49.41

Anſwer, { 62 Feet the Height.
49.41, or 49½ Feet the Diſtance from the ſecond Station.

NOTE. The Line BD is the Length of a *Scaling Ladder,* which would reach from the *Station* at D over the *Foſs* or *Ditch,* to the *Top* of the *Tower* at B.

PROBLEM

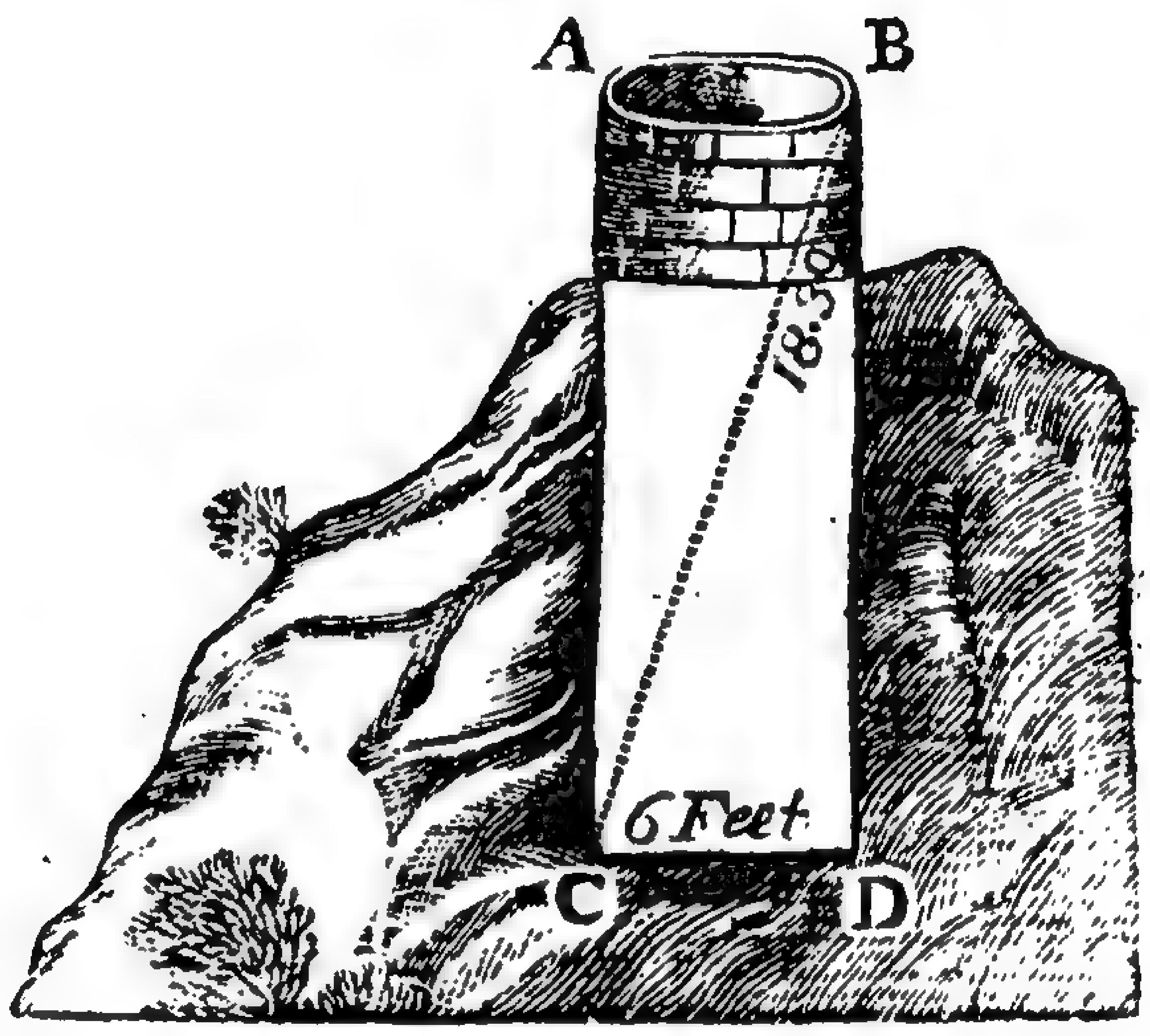
PROBLEM III.

To measure the *Depth* of a *Well*, or the *Height* of an *Object* from the *Top* of it.

First, look through the Sights of the *Quadrant* to the Bottom of the opposite Side the Well at C, so you will have the *Angle* CBD; next, take the *Breadth* AB at the Top, which is equal to CD at the Bottom: Then, by Case I. of Right Angle Triangles, you may easily find the *Depth* BD required.

Suppose the Angle at B, by Observation, to be 18° 30′, and the Breadth at the Top 6 Feet; what's the *Depth*.

```
                18.5
                185
                ----
                 925
                1480
                185
                ------
               342.25
                    3
               -------
              1.026.75
             57.3
Natural Rad. 58.32675 but 58.3 is enough.
```



(1st.) Find the Hypothenuse BC.

```
   ∠B    Op. Side  N. Rad.
As 18.5 — 6 —— 58.3
                  6
               -----
      18.5)349.8.(18.9 + the Line BC
           185
           -----
           1648
           1480
           -----
            1680
            1665
            ----
              15
```

(2d.) Find the Depth BD.

```
  To Hypoth. BC 18.9
   Add Side CD   6.
                ----
           Sum  24.9
Multiply by Differ. 12.9
                ----
                2241
                498
               249
               ------
Extract the Root 321.21(17.89 the Depth AB
               1
               ---
            27)221
               189
               -----
           348)3221
               2784
               -------
          3569)43700
               32121
               -----
               11579
```

Answer, { 17.89+, or 18 Feet, the Depth required.

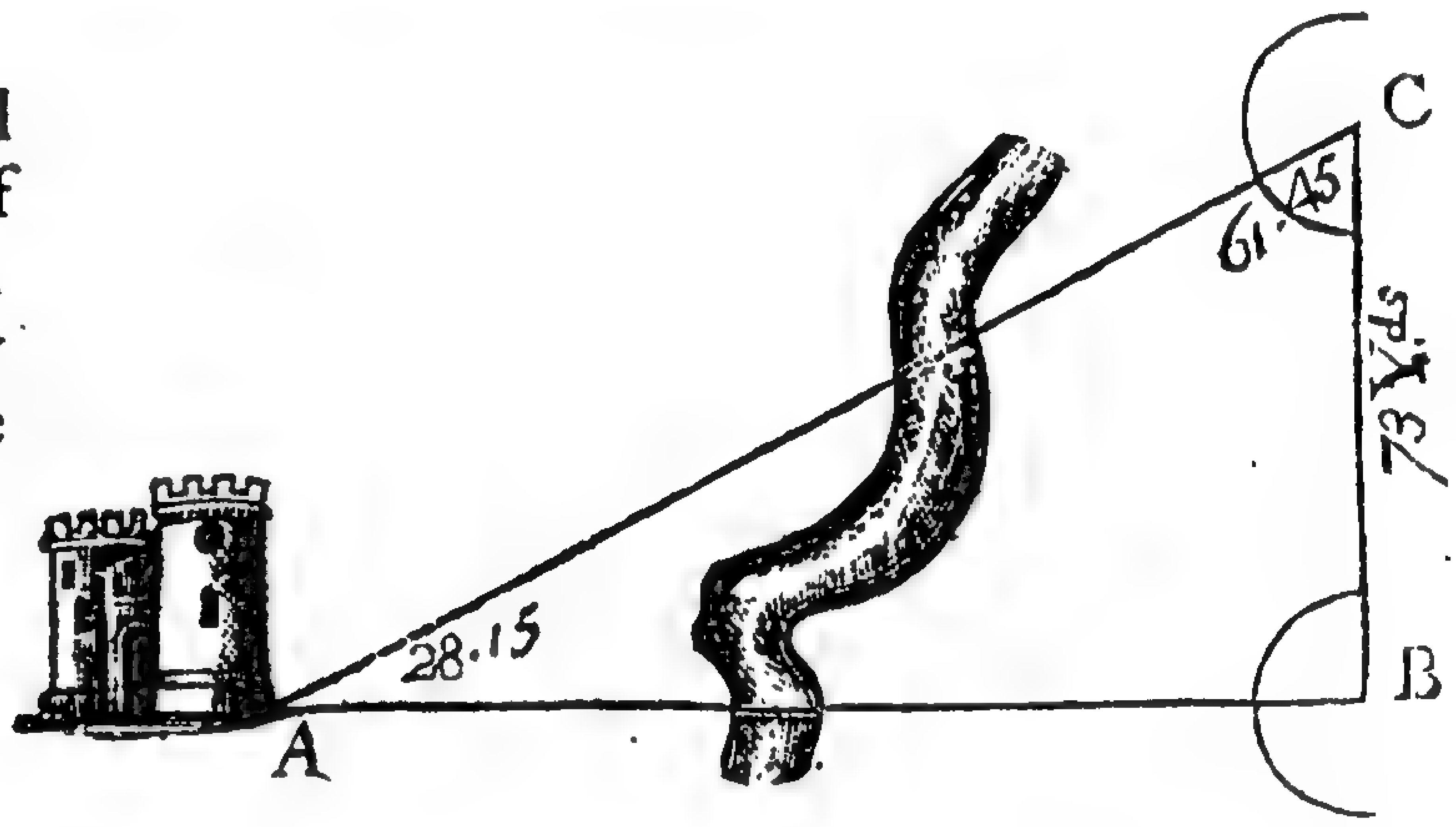
PROBLEM IV.

To meaſure the *Diſtance* of any *Object*.

Suppoſe yourſelf ſtanding at B, and a great Way off, as at A, you ſee a *Fort* or *Caſtle*, &c. or any other *Object*, whoſe *Diſtance* you would find from the Place where you now ſtand.

Firſt, a *Theodolite*, or *Semicircle*, being placed at B, lay the *Index*, with its Sights, on the *Diameter*, where the Degrees begin, and through them view the *Caſtle*, &c. at A. The Inſtrument remaining fix'd in this Poſition, move the *Index* to 90 Degrees, (that being a Right Angle) and view ſome Mark at a Diſtance, (the farther off the better) as at C.—Next meaſuring the Diſtance from B to C, which ſuppoſe 73 Yards, remove your Inſtrument, and ſet it up at C. Then, with the Index laid upon the Beginning of the Degrees, as before, turn the Inſtrument about, till you can ſee your *firſt Station* at B, where faſten it; then turn the Index till you can ſee the Object A, and obſerve what Degrees are cut, as ſuppoſe 61° 45′, which is the Quantity of the Angle where you ſtand; whoſe Complement to 90° is the Angle A.

Now, here are given all the Angles, and one Side of a *Right Angled* Triangle, to find either of the other Sides, which will be the *Diſtance* required.




```
      28.25
      28.25
      -----
      14125
     5650
   22600
   5650
   --------
   798.0625
          3
  ----------
 2.394.1875
 57.3
 ----
 59.69 or 59.7 Natural Radius
```

(1ſt.) Find the Diſtance from C.

Ang. A	: Op. Side BC	:: N. Rad.	: Diſt. CA
As 28.25	— 73	— 59.7	— 154.2

(2d.) Find the Diſtance from B.

```
To Hypoth. AC   154.2
 Add Side BC     73
                -----
          Sum   227.2
Multiply by Differ.  81.2
                -----
                 4544
                2272
               18176
               ------
Extract the Root 18448.64(135.8+, Diſt. from B
               1
               --
            23)84
               69
               ----
           265)1548
               1325
               -----
          2708)22364
               21664
               -----
                 700
```

Anſ. yards { 135.8+, or 136 Diſt. from B. / 154.2, or 154¼ Diſt. from C. }

PROBLEM

PROBLEM V.

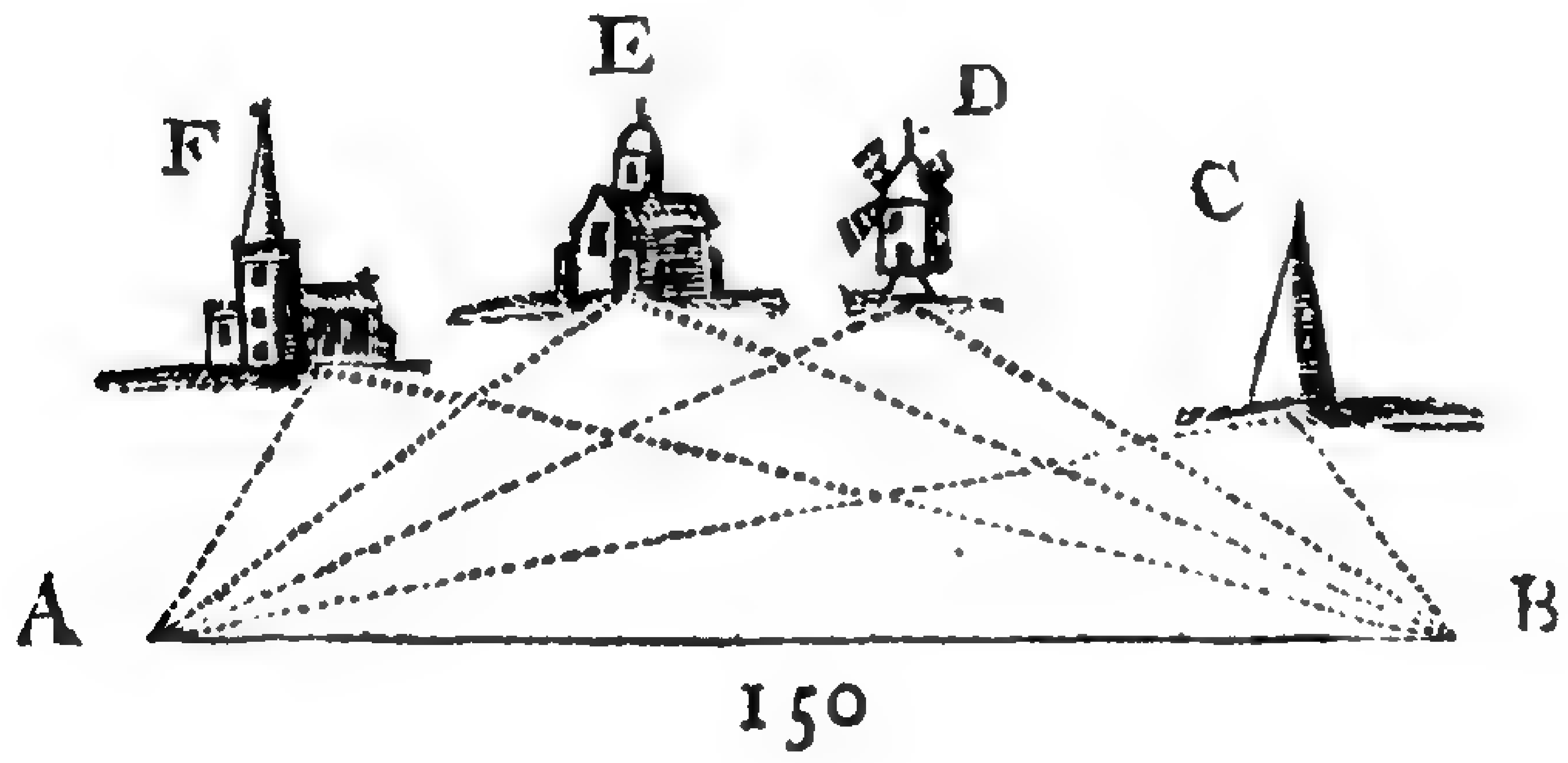
To take the *Diſtances* of ſeveral *inacceſſible Objects*, as *Forts*, *Churches*, in a *Town*, or *Squadron of Ships* at *Sea*, and to delineate them upon *Paper*.

Firſt, make choice of two Places, from either of which you may conveniently ſee all the *Objects*; which two Places let be A and B in the following Figure.—This being done, ſet up your Inſtrument at A, laying the *Index* on the *Diameter*, and turn the whole Inſtrument about, till, through the Sights, you ſee your ſecond Station B. Then, fixing the Inſtrument, direct your Sights to the ſeveral *Objects*, C, D, E, and F; noting down the Degrees cut at each Obſervation, which ſuppoſe to be as in the Table.

Then, remove the Inſtrument to B, laying the *Index* on the *Diameter*, and turn it about, till, through the Sights, you ſee your former *Station* at A; then direct your Sights to every one of the Objects at C, D, *&c.* ſetting down the Degrees at each Obſervation, as in the Table. Alſo meaſure the *Stationary Diſtance*, and ſet that down.

	C	D	E	F
1ſt Station	11	23	36	59
2d Station	51	31	22	13
Stationary Diſtance 150 Yards				

Firſt, upon a Piece of Paper draw the Line AB; and from a *Scale of equal Parts*, take off, with your Dividers, the *Stationary Diſtance* = 150, and ſet it from A to B, ſo will A repreſent your *firſt Station*, and B the *ſecond*. Then lay the *Center* of the *Protractor* upon the Point A, with its Diameter upon



the Line AB; keeping it faſt, make Marks by the Edge at 11, 23, 36, 59, and draw Lines from the Point A through each of thoſe Marks. Then upon B place the *Center* of your *Protractor*, its Diameter lying upon the Line AB; make Marks by the Side at 13, 22, 31, 51. Then draw Lines from the Point B through each of theſe Marks, and where the Lines cut the former *correſpondent Lines* there will be found the Places repeſenting theſe *Objects*. Then any of theſe Lines being taken in a Pair of Dividers, and applied to the Scale you laid your *Stationary Diſtance* down by, will give you their *Diſtances*, either from your *Stations* or from one another.

The *Diſtance* of any of theſe *Objects* from *either Station*, &c. may be found by *Calculation*; *one Side* and the *Angles* being given: But I ſhall omit that, on purpoſe to *exercise* the *Learner*'s *Genius*, and proceed.

PROBLEM

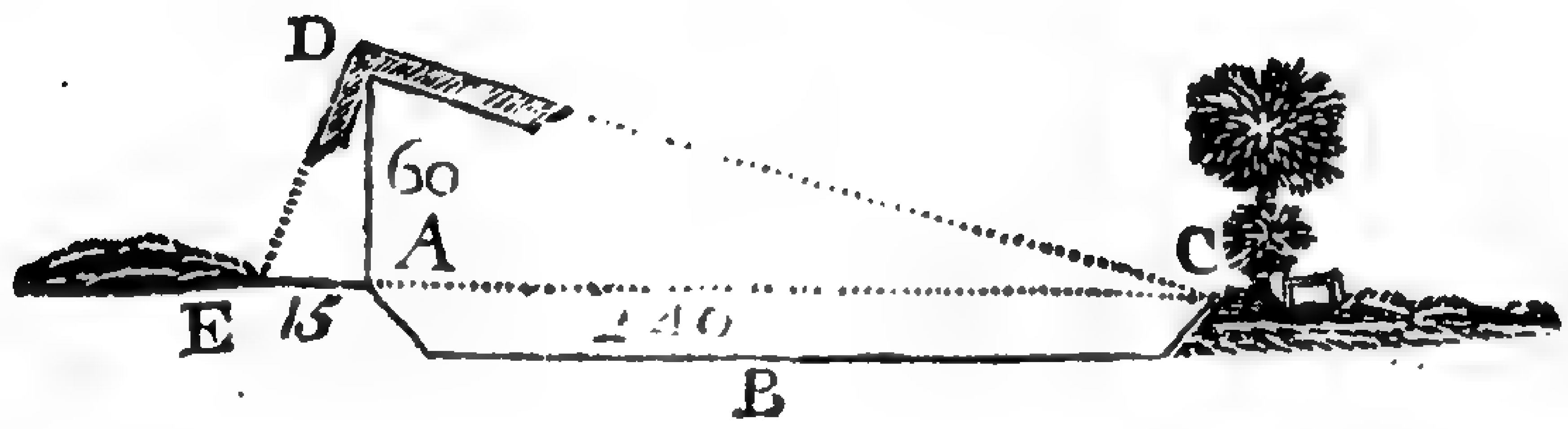
PROBLEM VI.

To take the *Distance*, upon level Ground, of any *inaccessible Tree*, *Fort*, &c. or *Breadth* of a *River*, by a *Common Square*.

Suppose there is a River, as A B C, whose *Breadth* you want to know. —First, upon the Bank, at A, set up a Stick, AD, which suppose to be 5 Feet, or 60 Inches high; then fixing your *Square* on the Top, at D, look by the Side of it till you see the Edge of the opposite Shore C, and fasten it, as it may not go from that Position. This done, extend a Thread from D, by the other Side of the *Square* till it touch the Ground at E. Then measure the Distance EA, which suppose 15 Inches, (or 1 Foot 3 Inches) and you may find AC (by Reason of similar Triangles) thus.

	Dist. EA	:	Side DA	: :	Side DA	:	Dist. AC
As	15	——	60	——	60	——	240 Inches,

which, reduc'd to Feet, give 20 for the Breadth of the *River* sought.



NOTE. There are various Ways of taking *Heights* and *Distances*; but the *best* is to take the Angles for *Heights* by a *Quadrant*; and the Angles for *Distances* by a *Semicircle* or *Theodolite*; and calculate by the foregoing *Axioms*. In all *Heights* the *Triangle* stands *upright*; but in *Distances*, it is suppos'd to lie *flat* or *horizontal*.

☞ I shall here shew the Learner, how to take the *Breadth* of a *River*, or a *small Distance*, without any *Instrument whatever*; which is thus. Standing upon the Bank, bring down the Edge of your *Hat*, till it appears to touch the opposite Side, then steady your Head by laying your Hand under your Chin, and turn yourself towards some *level Ground*, observing where the Edge of your *Hat* glances upon it; for then, the *Distance* from you to that *Place*, is *equal* to the *Breadth* of the *River*, or *Distance* required.

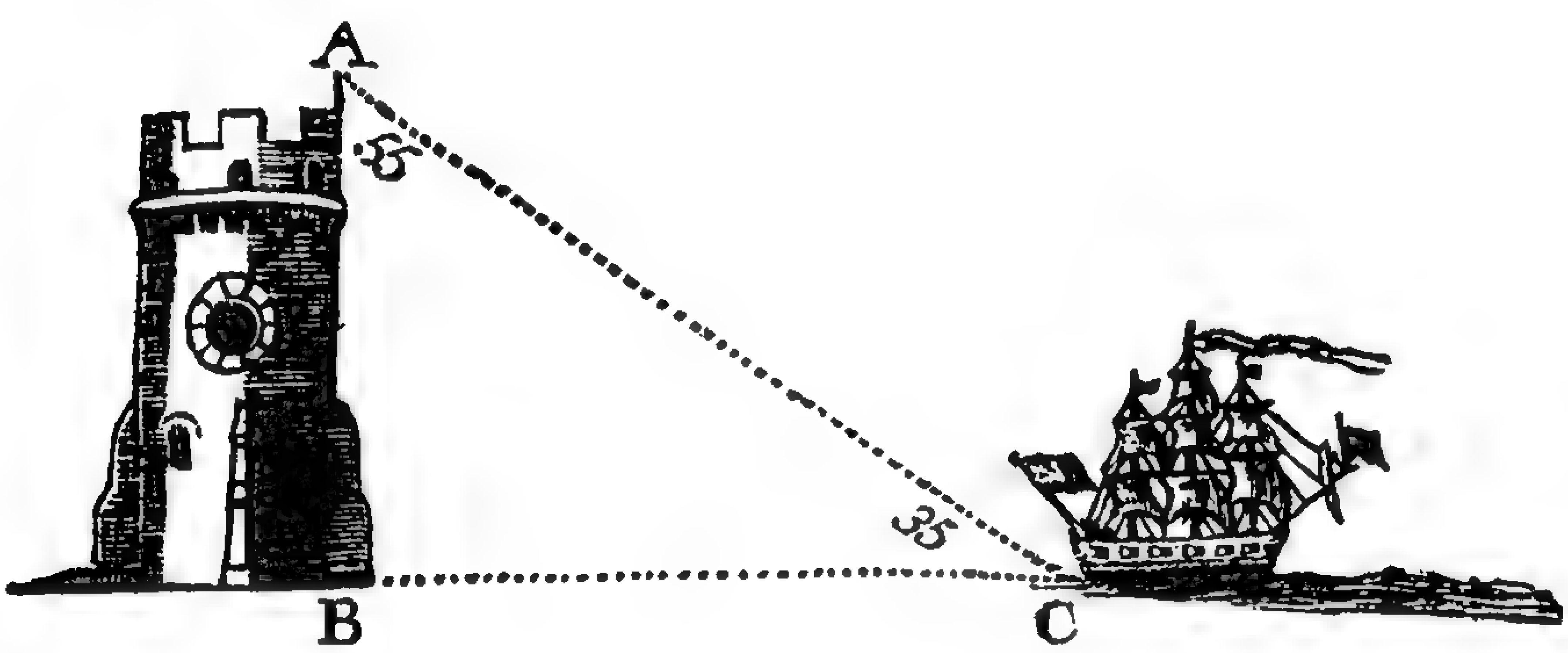
PROBLEM

PROBLEM VII.

To find, from the Top of a *Fort*, or *Tower*, how far any *Tree*, *Ship*, &c. is from you.

Let A be the Top of a *Tower* or *Castle* standing by the *Sea-side*; and let C be a *Ship* at Sea, or lying at Anchor, and you would know how far that Ship is off the *Castle-wall*.

With your *Quadrant* or *Semicircle*, direct your Sights from the Top of the *Tower* to the Place where the *Ship* is, and take the Angle, which we will suppose to be 55 Degrees. Then the *Castle-wall* being known before to be 143 Feet high, you may easily find the *Distance* of the *Ship* from the *Wall* in this Manner.



(1st.) Find the Side AC.

Ang. C	: Height Wall	:: Nat. Rad.	: AC
As 35	—— 143	—— 61	— 249.2

(2d.) Find the Base BC.

N. Rad.	: AC	:: Ang. A	: Dist BC
As 67	—— 249.2	—— 55	— 204.56

By this Method you may easily discover if a *Fleet* of *Ships*, or one *single Ship*, at Sea, makes towards you or not. For having observ'd from the Top of the *Fort* the Angle from thence to the *Ship*, and noted it down, rest a little Time, and observe again: Then, if the Angle be *bigger* than before, the Ship is *departing* from you; but if *less* she is *making towards* you.

PROBLEM VIII.

To take the *Perpendicular Height* of a *Hill* or *Mountain*, and alſo the *Horizontal Line*, or *Baſe*, on which it ſtands.

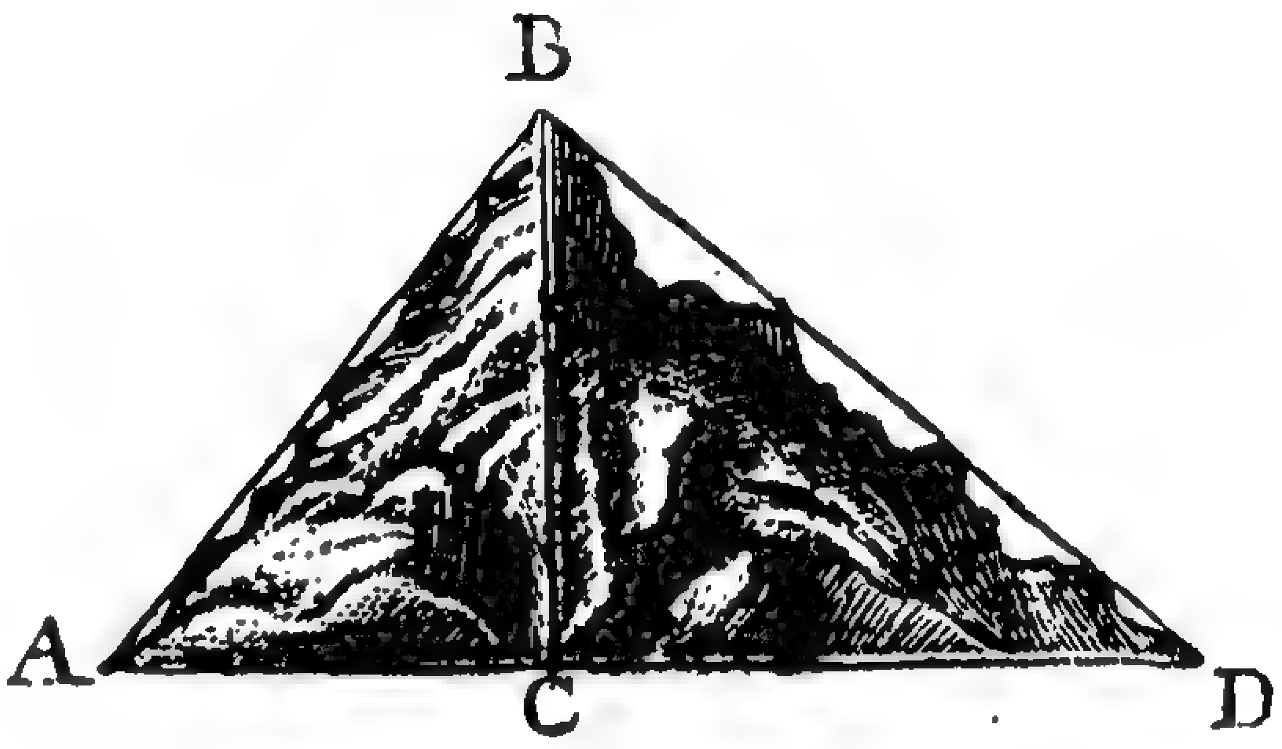
Let ABD be the *Hill*.—Firſt, ſet up a Mark on the Top at B, equal to the Height of the *Quadrant* or Inſtrument that is uſed at the Bottom, from whence you intend to make your Obſervation. Then by looking through the Sights to B take the Quantity of the Angle at A, which we will ſuppoſe to be 50°. Next meaſure the Hill from A to B, which let be 546 Feet. This being done, you may eaſily find the *Perpendicular* BC, or Part of the *Baſe* AC, by Caſe II. of Right Angles.

For the Perpendicular BC.

N. Rad. : Side AB :: Ang. A : Perp. BC
As 65.2 — 546 — 50 — 418.7

For the Side AC.

N. Rad. : Side AB :: Ang B : Side AC
As 62.2 — 546 — 40 — 351.1



Now, as the Hill *deſcends*, you may go on the oppoſite Side, and make the like Obſervations, viz. ſet up the Inſtrument at D, and take the Angle D, 40°, and meaſure the Side DB, 651 Feet, then you may find the Side CD in the ſame manner you did AC. Thus,

Nat. Rad. : Side DB :: Ang. B : Side CD
As 65.23 — 651 — 50 — 499.

If to the Part AC = 531.1 Feet, be added the Part CD = 499 Feet, the Sum 850.1 Feet will be the whole Length of the *Horizontal Line* AD requir'd.

The *Perpendicular Height* DC is = 418.7 as above.

PROBLEM

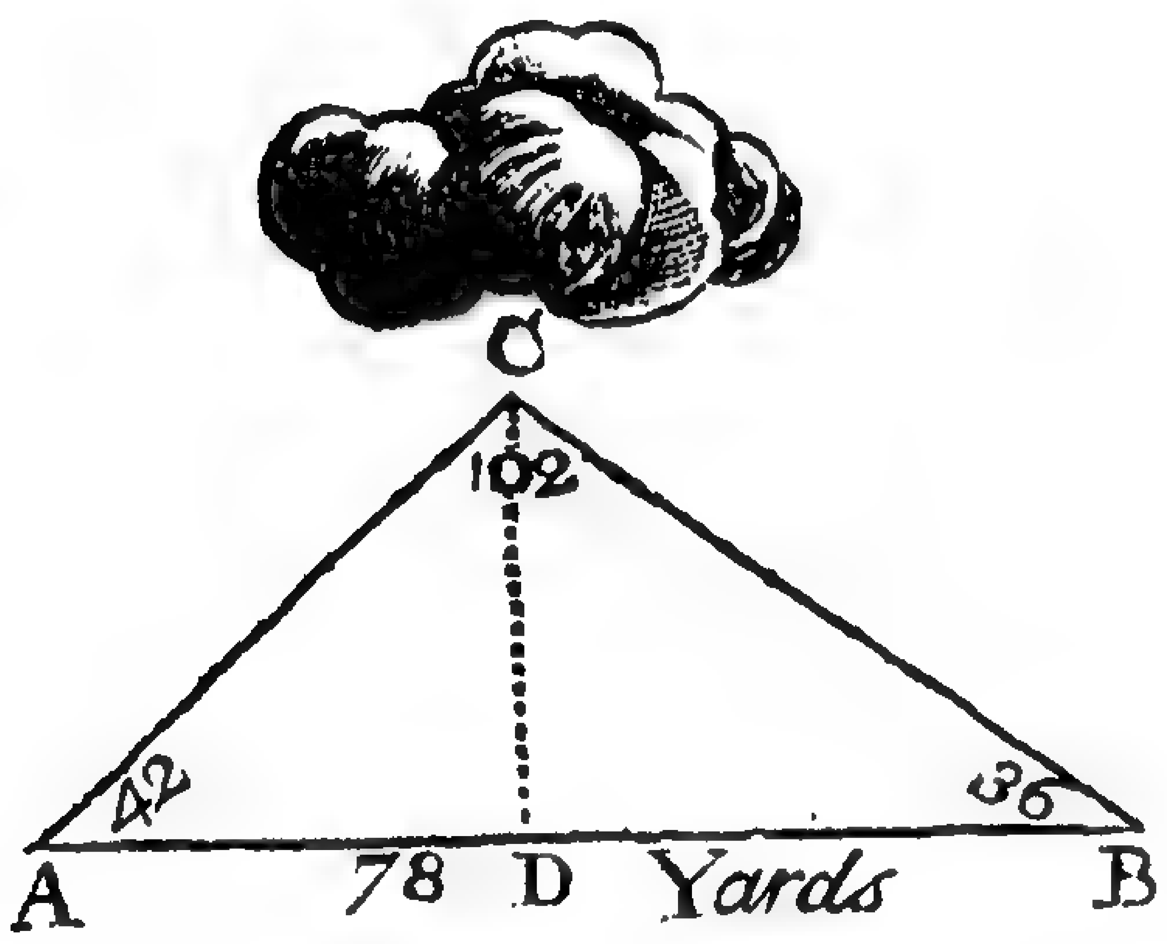
PROBLEM IX.

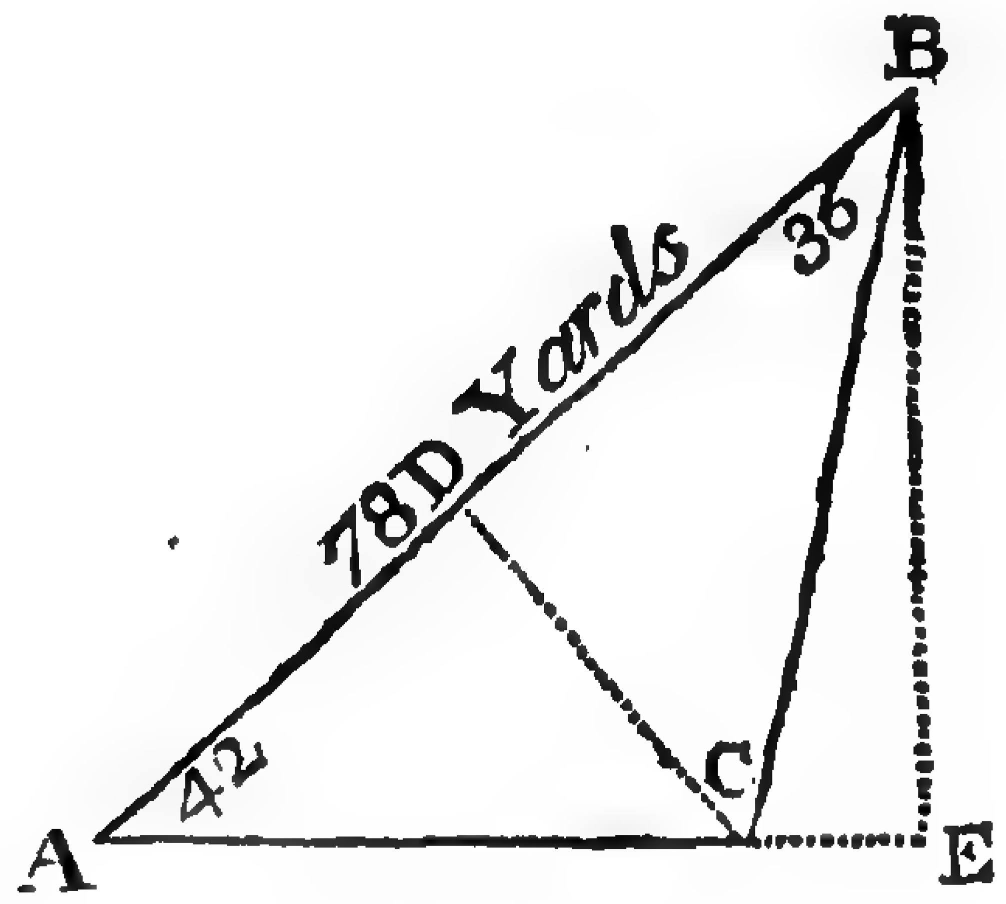
To take the *Height* and *Distance* of a *Cloud.*

Suppose it was requir'd to find the Height of the *Cloud* C.

Let a Person standing at A, look through the *Quadrant* to the *Cloud* at C, so will the Thread cut the Angle at A. At the same Time let another Person, making the like Observation at B, take the Angle B. Then measure the *Distance* between the *two Stations*. By this means you will have *one Side* and *all* the *Angles* of an *Oblique Angle Triangle* given, from whence you may easily obtain the rest, and particularly the *Perpendicular* CD, which will be the *Height* of the *Cloud* requir'd.

EXAMPLE. Suppose the Angle at A, by Observation, be 42°, the Angle B 36°, and the Distance AB 78 Yards: I demand the *Height* of the *Cloud.*





(1st.) Find the Perpendicular BE in Triangle ABE *.

N. Rad. : Op. Side AB : : Ang. A : Perp. BE
As 62.6 —— 78 —— 42 —— 52.1

(2d.) Find the Hypothenuse CB in Triangle CBE.

Ang. BCE : Op. Side BE : : N. Rad. : Hyp. BC
As 78 —— 521 —— 79 —— 53.2

(3d.) Find the Perpendicular CD in Triangle CDB.

N. Rad. : Op. Side BC : : Ang DBA : Perp. CD
As 61.1 — 53.2 — 36 — 31.3

Answer, 31.3 Yards, the Height requir'd.

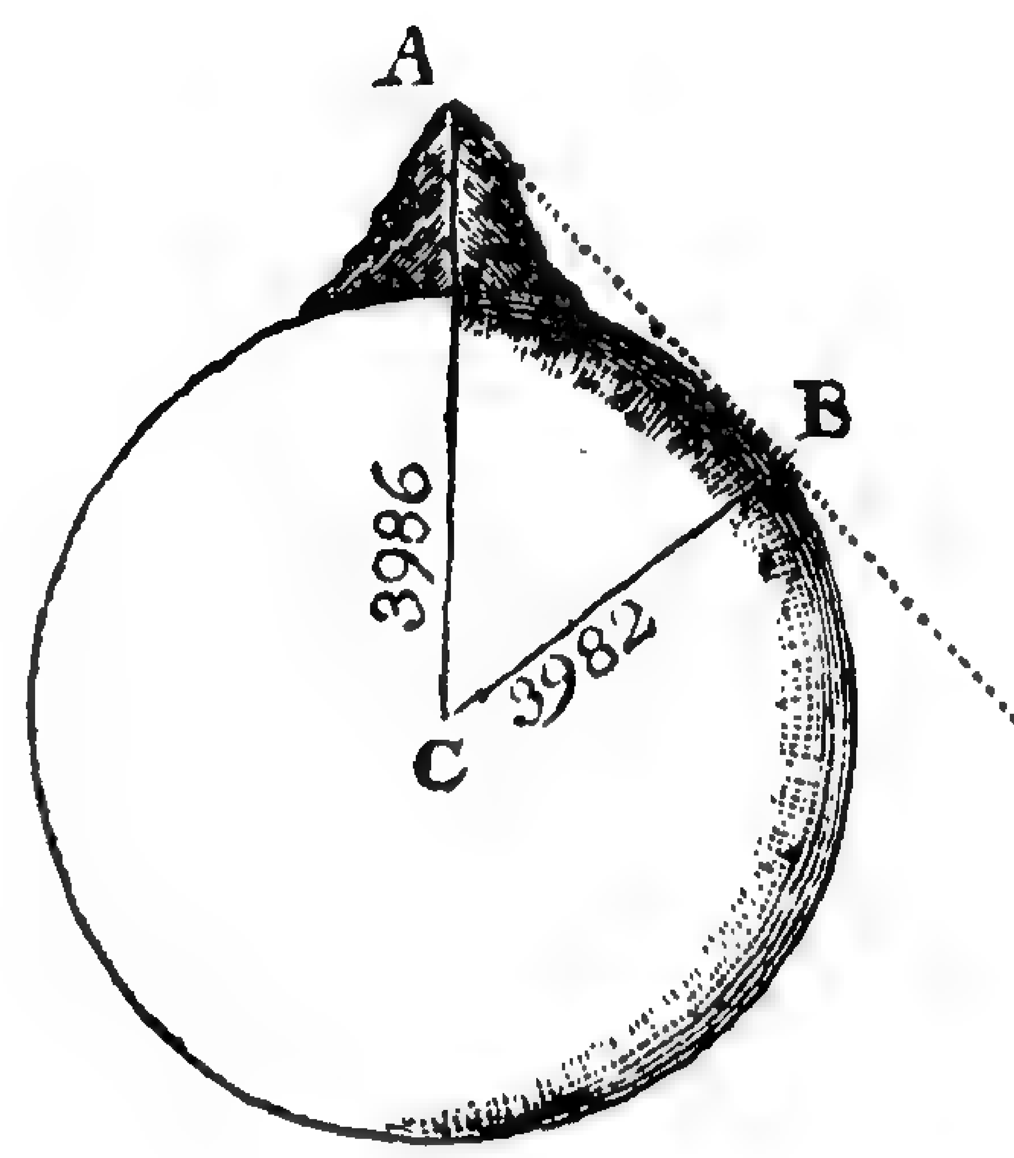
* The Figure on the *Right Hand* is only that on the *Left* set in a different Position, to shew in a more ~~natural or~~ easy Manner, how the Perpendicular falls from the End of the given Side AB, upon the Side AC produc'd to E.

PROBLEM

PROBLEM X.

To find how far a *Hill* of any given *Height* can be ſeen at *Sea*, or upon *level Ground*.

How far, for Inſtance, can the *Pike* of *Teneriff* be ſeen at *Sea*, whoſe Height is about *four Miles*.



☞ The *Circumference* of the *Earth* is ſuppos'd by *Mathematicians* to be divided into 360 equal Parts, called *Degrees*; and our countryman, Mr. *Norwood*, has found, by meaſuring from the *Tower* of *London* to the Middle of the City of *York*, in the Year 1635, that one of thoſe Degrees, upon the Earth's Surface, contains $69\frac{1}{2}$ Miles; according to which Meaſure, we find the Earth's *Circumference* to be 25020 Miles—its *Diameter* 7964—and its *Semidiameter* 3982.

Then in the Triangle ABC, Right Angled at B, we have the Side CB = the Earth's Semidiameter 3982.—Alſo the Line AC = the Semidiameter and Height of the Mountain together = 3986.—To find AB, the Diſtance from the Hill to the viſible Horizon.

To Hypoth. AC 3986
Add Leg BC 3982

Sum 7968
Multiply by Differ. 4

Extract the Root 31872.00(178.5 The Diſtance the Mountain can be ſeen.
1

27)218
189

348)2972
2784

3565)18800
17825

975

This *Mountain* can be ſeen 178.5 Miles at *Sea*.

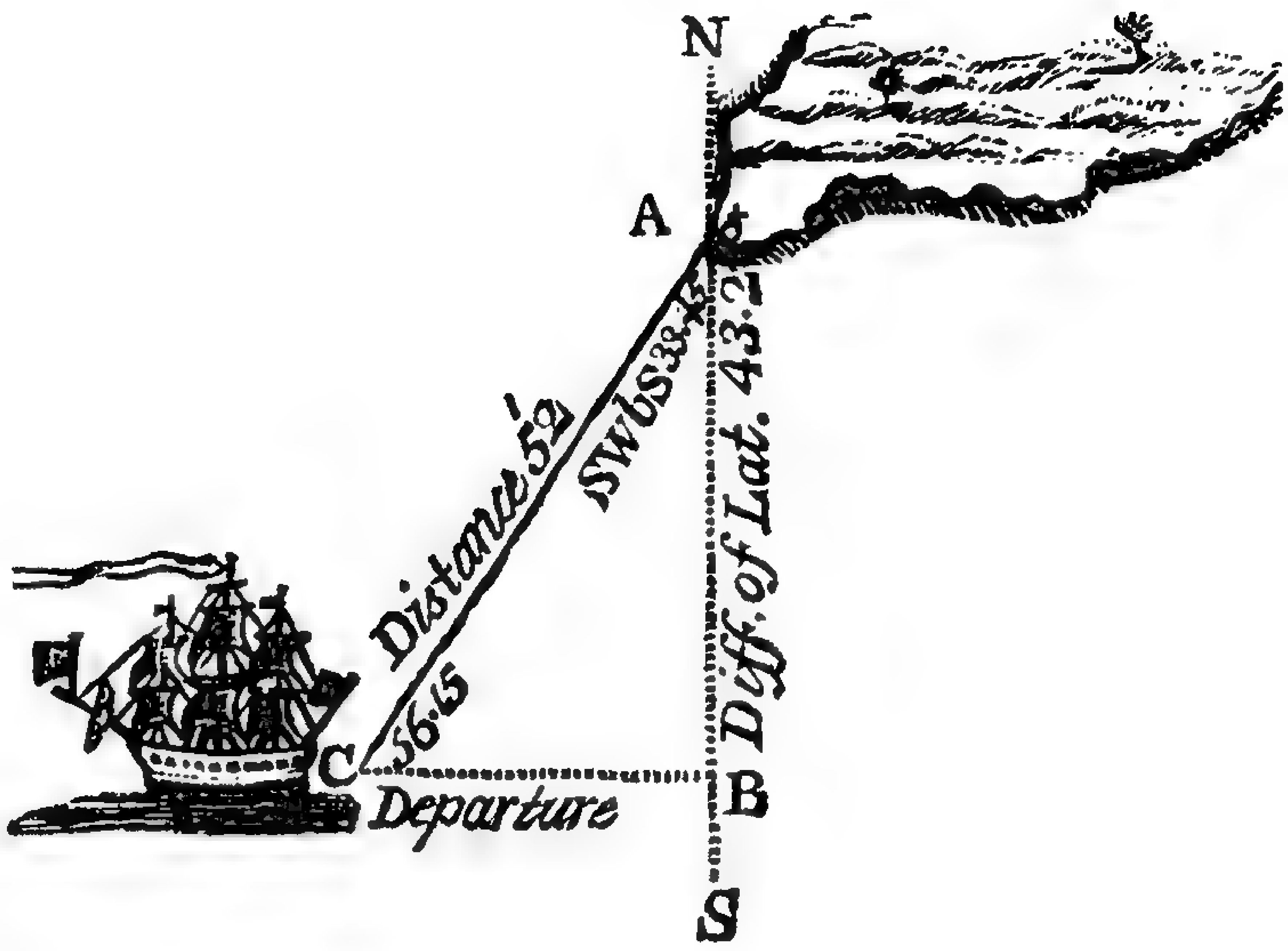
PROBLEM

PROBLEM XI.

The *Distance run* at Sea, and the *Course*, given; to find the *Difference* of *Latitude* and *Departure* from the *Meridian*.

Suppose a *Ship* from A, in the *Latitude* of 50° North, sails away, SW by S. 52 Miles, to C: I demand the *Latitude* she is in, and also her *Departure* from the *Meridian*.

```
        33.75
        33.75
        -----
        16875
       23625
      10125
     10125
     ----------
     1139.0625
             3
     ----------
     3.417.1875
    57.3
    ----
Nat. Rad. 60.7 is enough.
```



(1st.) For the Departure.

```
 N. Rad.  Hyp. AC  ∠ A
As 60.7 —   52  — 33.75
                    52
                  -----
                  6750
                 16875
                 -----
          60.7)1755 00(28.9  The Departure
               1214                CB
               ----
               5410
               4856
               ----
                5540
                5463
                ----
                  77
```

```
Latitude departed from      50° North
Difference of Latitude       0   43.2
                            ---------
Latitude the Ship is in     49   16.8
```

(2d.) For the Diff. of Latitude AB.

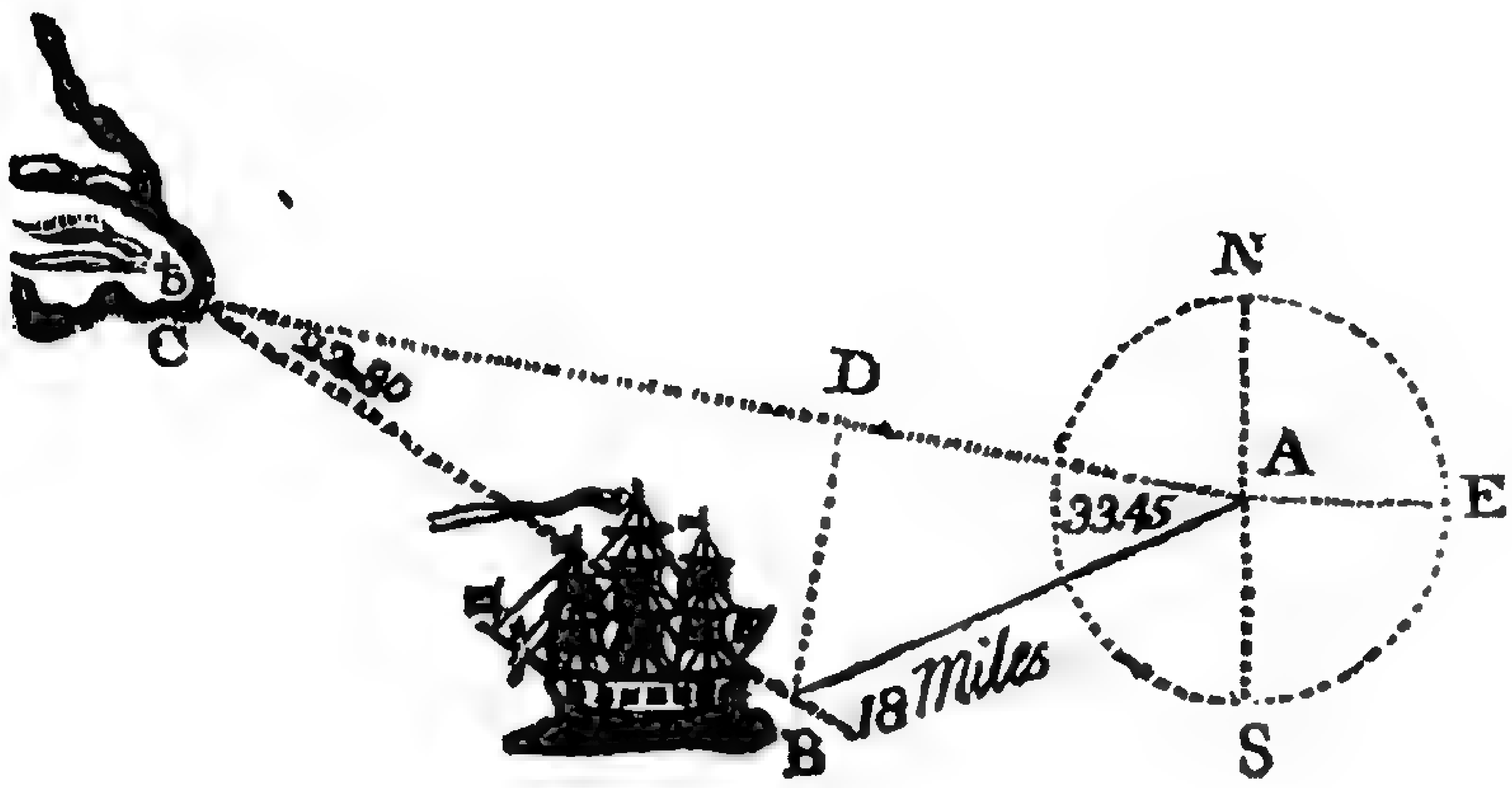
```
      To Hypoth. AC  52
       Add Side BC  28 9
                    ----
               Sum  80.9
Multiply by Differ.  23.1
                    ----
                     809
                    2427
                   1618
                   -------
Extract the Root   1868.79(43.2  Diff. of Latitude
                   16
                   ---
                 83)268
                    249
                    ---
                862).979
                    1724
                    ----
                     255
```

NOTE. That in all Cases of *Sailing*, we suppose the *Top* of the Book *North*, and *Bottom South*; the *Right Hand East*, *Left Hand West*.—The *Distance run* is the *Hypothenuse*; the *Difference of Latitude* is the *Perpendicular*; the *Departure* the *Base*. The *Angle* at the *Perpendicular* is the *Course*, and the *other* its *Complement*.

PROBLEM XII.

To take the *Distance* of any *Cape, Fort,* or *Island,* from a *Ship* at *Sea.*

Sailing W. S. W. I saw, at some Distance, a Point of Land, which I *set*, and find it bears from me W. by N. and having sailed 6 Leagues further, I find it then bears from me N. W. by W. I would know how far this Land is from me.



(1st.) Find the Perpend. BD in Triangle ABD.

```
  33.75
  33.75
  -----
  16875
 23625
10125
10125
---------
1139.0625
        3
---------
3.417.1875
57.3
Nat. Rad. 60.7 is enough.
```

```
N. Rad.   Op. Side AB   ∠ A
As 60.7   —   18   —    33.75
                           18
                        -----
                        27000
                        3375
                        -----
              60.7)607.50(10 Perpend.
                   607
                   ---
                   .50
```

(2d.) Find the Distance CB in Triangle BCD.

```
  22.5
  22.5
  ----
  1125
 450
450
------
506.25
     3
------
1.518.75
57.3
Nat. Rad. 58.8 is enough.
```

```
∠ C      Op. Side BD   Nat. Rad.
As 22.5  —   10   —    58.8
                         10
                       ----
              22.5)5880(26.13
                   450
                   ----
                   1380
                   1350
                   ----
                    300
                    225
                    ---
                    750
                    675
                    ---
                     75
```

Answer, 26.13 Miles, the Distance requir'd.

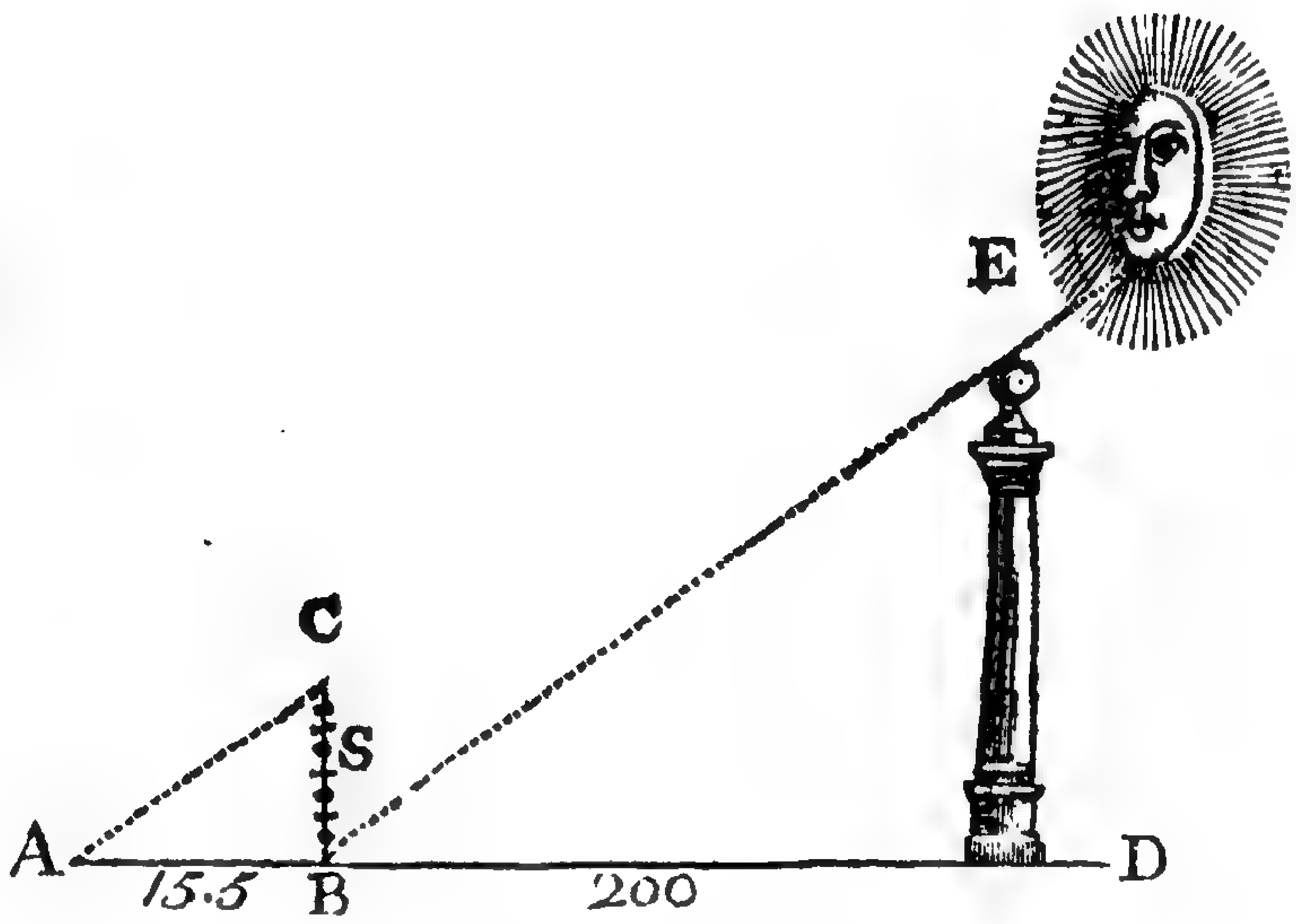
PROBLEM

PROBLEM XIII.

To take the Height of a *Tree, Fort, Obeliſk, Pyramid,* or any *Object,* by a *common Stick* only, when the *Sun* or *Moon* ſhines upon it.

Take a Stick of any Length, ſuppoſe 8 Feet; ſet it upright upon the Ground, as at CB in the Figure below. Mark the End of its Shadow at A, and meaſure its Length from B to A, which ſuppoſe to be 15.5 Feet. Then meaſure the Length of the Shadow of the *Pillar* or *Obeliſk* BD, which ſuppoſe to be 200 Feet. This being done, you may eaſily find the Height: For (by Reaſon of like Triangles) it will always hold,—as the Length of the Shadow of the Stick AB in the ſmall Triangle, is to its Height CB; ſo is the Length of the Shadow of the Obeliſk BD in the great Triangle, to DE the Height thereof.

Shad. St. Stick. Shad. Ob.
As 15.5 — 8 — 200
8
———
15.5)1600.0(103.2 Height requir'd
155..
———
..500
465
———
.350
310
———
.40



In this *Manner* the Heights of the *Pyramids* in *Egypt* have been taken. Thoſe ſtupendous Buildings are ſuppoſed to have been erected by the *Children of Iſrael,* when in Bondage, for *Sepulchers* for the *Egyptian Kings.* They are the greateſt Pieces of *Antiquity* now in Exiſtence. There are ſeveral *ſmaller Ones,* but the *largeſt,* which is juſtly eſteemed one of the *Wonders of the World,* is 500 Feet in *Perpendicular Height;*—700 Feet if meaſured *obliquely* from the Bottom to the Top;—and its *Baſe* covers about 11 *Acres of Ground.*

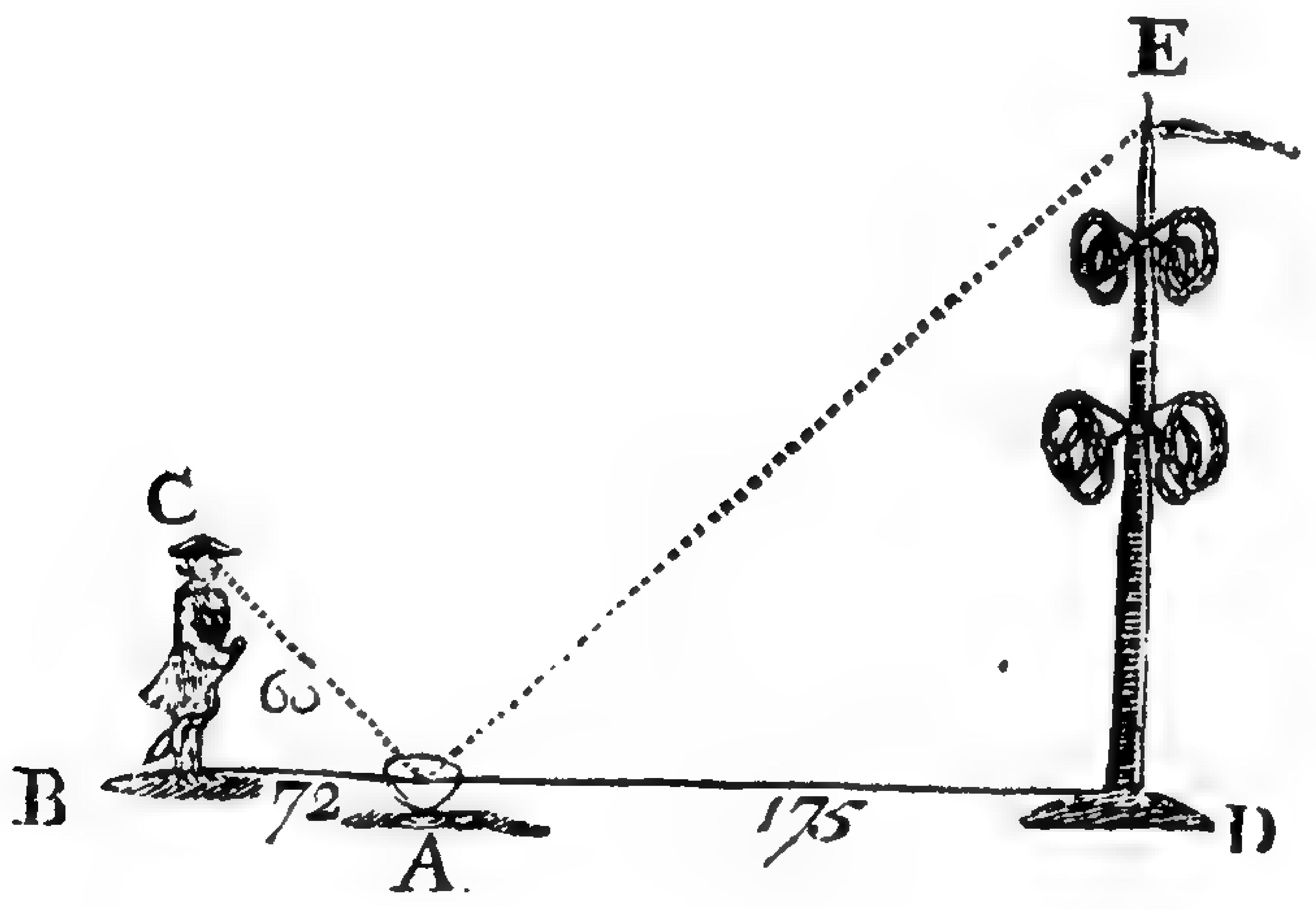
PROBLEM

PROBLEM XIV.

To take the *Height* of any *acceſſible Object* by a *Baſon of Water*, or common *Looking-glaſs*.

Travelling along the Road, I ſee a fine *May-pole*, whoſe Height I would gladly know; but having no *Mathematical Inſtrument* with me, I procure a *Baſon of Water*, which I ſet upon the Ground, at ſome Diſtance from the Pole, as at A; then I go backwards, till I ſee the Top of the Pole in the Middle of the Water, as at B. This done, I meaſure the Diſtance from my Station at B to the Baſon at A, which ſuppoſe 72 Inches; and alſo meaſure from the Baſon to the Bottom of the Pole at D, and find it 175 Inches. Next I meaſure the Height of the Eye from the Ground, which ſuppoſe 60 Inches. Then ſay, by the *Rule of Three*,

As the *Diſtance* from my *Station* to the *Baſon*,	= 72	Inches.
Is to the *Height* of the *Eye*,	= 60	
So is the *Diſt*. from the *Baſon* to the *Foot* of the *Pole*,	= 175	
To the *Height* of the *Pole* requir'd,	= 145.8	



The ſame Thing may be obtain'd by a *Looking-glaſs*, laid truly *Horizontal*, or level on the Ground, by walking back till you can ſee the *Top* of the *Building*, &c. in the *Middle* of it, as was done by the *Water*.

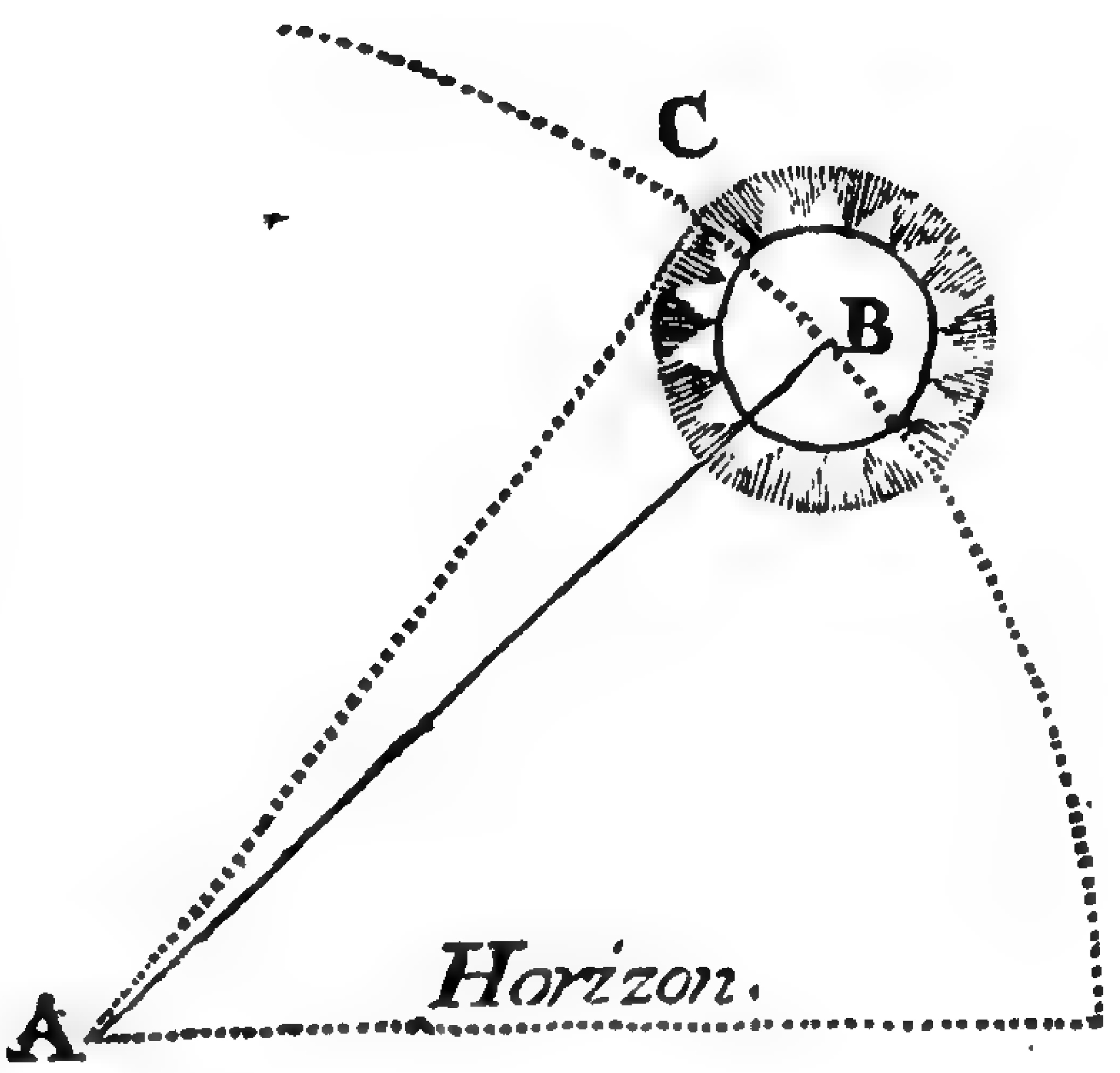
PROBLEM

PROBLEM XV.

To take the Diſtance of the *Sun*, *Moon*, or any of the *Heavenly Bodies*.

Suppoſe it was requir'd to find the Diſtance of the *Sun*, in *Diameters* of his Body from us.

With a *Quadrant* nicely graduated, take the *Altitude* of the *lower* and *upper Limb* in *Degrees* and *Minutes*, and ſubtract the one from the other; the Remainder will give the *Diameter* of the *Sun*, which we will ſuppoſe, in this Caſe, to be 32 Minutes.



Then have we given in the Triangle ABC, Right Angled at B,—the Angle at A = 16′, and the Side CB = .5 = Half the Diameter of the Sun, whoſe whole Diameter we will call 1; to find the Diſtance, or Side AC.

Ang. A : Side BC :: Nat. Rad.
As .26 —— .5 —— 57.3
.5
.26)2865(110 Diameters; and ſo far is the *Sun* of its own *Breadths* from us in the *Winter* *, but in *Summer*, the Angle being a little ſmaller, he muſt, conſequently, be a little further from us.
26
26
26
05

Having found the *Diſtance* of any *Heavenly Body* in its own *Diameters* from us; you may eaſily tell its *Diſtance* in *Miles*, if you firſt know the *Diameter* of that Body in *Miles*. For, the Diſtance in Diameters, multiply'd by the Miles in one Diameter, gives the Diſtance ſought.

For the Uſe of the Learner, I have here ſubjon'd a Table of the *Diameters* of all the *Planets* in *Engliſh Miles*; whoſe *Diſtances* he may calculate at his Leiſure.

Sun 800.000—*Mercury* 2460—*Venus* 7906—*Earth* 7964—*Mars* 4444 *Jupiter* 81.155—*Saturn* 67.870—*Moon* 2175.

* In this Manner we can tell the *apparent* Diſtance of any of the *Heavenly Bodies*: For the *Sun* appearing about 1 *Foot* in Diameter, his *apparent* Diſtance can be only 110 *Feet*, or 37 *Yards*. The *apparent* Diſtance of the *Moon* is nearly the ſame.

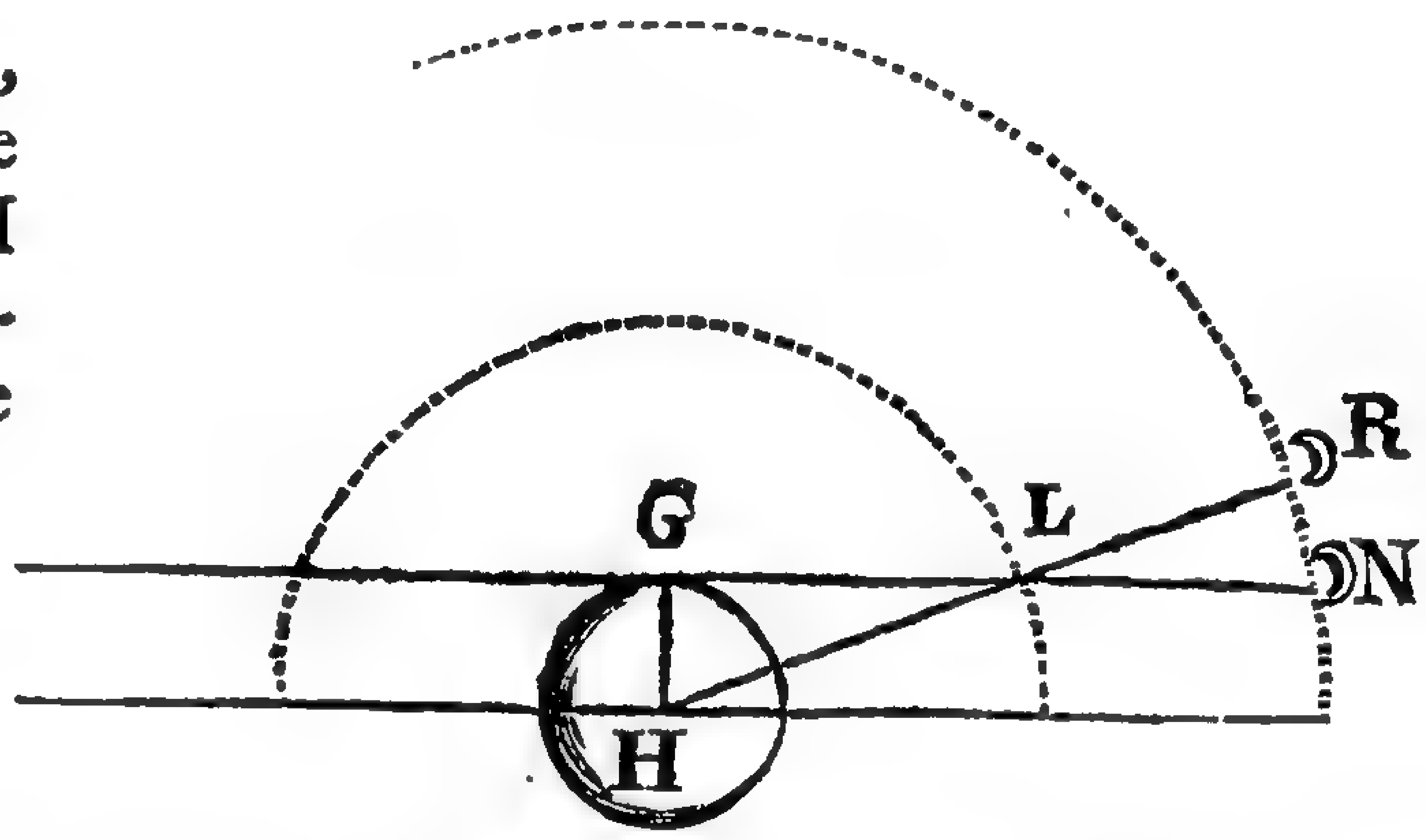
PROBLEM XVI.

To find the *Distance* of the *Moon* from the *Earth* another Way.

As the Places of all the *Heavenly Bodies* are computed from the *Center* of the *Earth*, but we are obliged to view them from the *Superfices*, they must therefore appear something lower in the Heavens than they really are.

Thus, suppose, for Instance, the ☽ to be at L in the visible Horizon; an Observer at G will see her in the Line GN, but an Eye in the Center will see her in the Line HR.—The former is her *apparent Place*, known by Observation with exact Instruments; the latter her *true Place*, and known by *Calculation* from *Astronomical Tables* of her Motion. The Difference between these two Places is the Measure of the Angle GLH, which is called the *Horizontal Parallax*. This Angle has been found, when the ☽ was at a *mean Distance*, to be 57 Minutes nearly.

Then in the Triangle HGL, Right Angled at G, we have the Angle at L = 57′, the Side GH = the Earth's *Semidiameter* given; to find HL, which is done thus.



Ang L : Semid. GH :: N. Rad.
As .95 — 1 — 57.3
1

.95)57.30(60 Semidiameters: which multiply'd by the Earth's Semidiameter=4000 nearly, you have 240000 Miles the Distance of the Moon sought.
570

30

When the ☽ is at her *least* or *greatest* Distance from the Earth, she will be about 3 *Semidiameters* of the Earth *nearer* or *farther off*, than at her *mean* Distance.

In like Manner may the Distance of any of the *Planets*, *Comets*, or other *cælestial Phænomenon* be determined, by obtaining its *Parallax* in the *Horizon*.

NOTE. To find the *Distance* of any Place to which the *Sun*, *Moon*, or any *Star* is *vertical*, i. e. over its Head, or in its *Zenith*. See my *Geography*, p. 30.

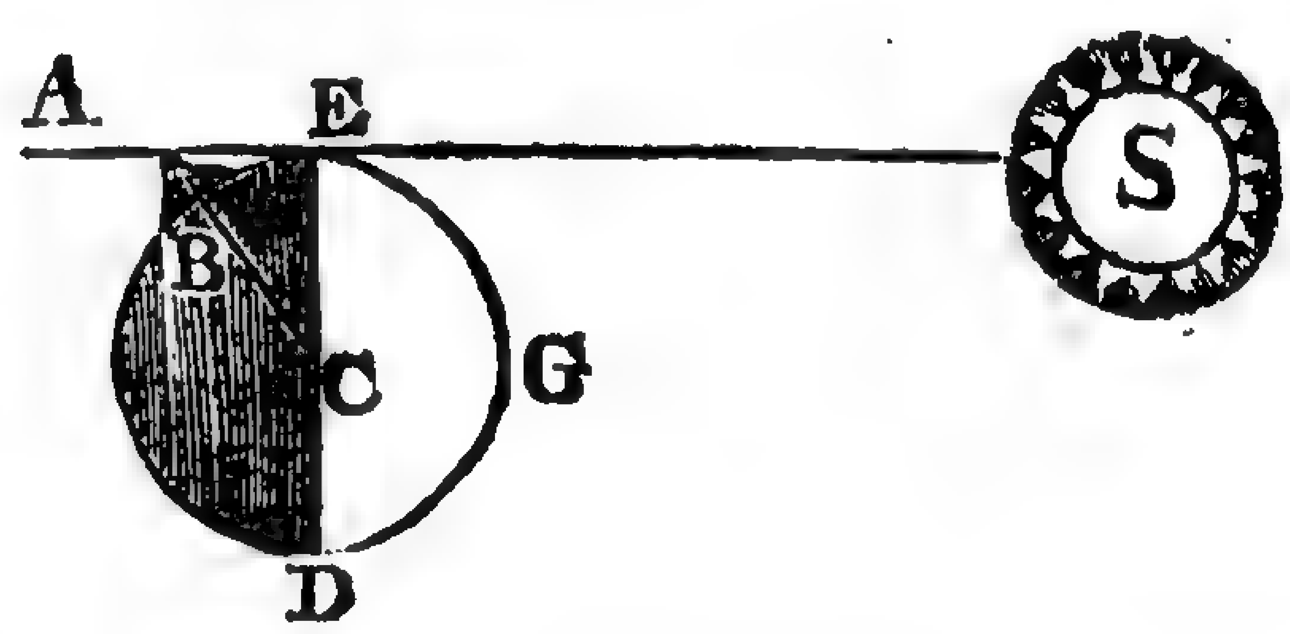
PROBLEM

PROBLEM XVII.

To meaſure the *Height* of a *Mountain* in the *Moon*.

The *Moon* is ſuppos'd to be compos'd of *Land* and *Water* as our Earth is; conſequently there muſt be ſome Unevenneſſes, or Inequalities, of *Hills* and *Vallies* as here. This indeed is confirmed by viewing her through a good *Teleſcope*; for then we find, that the Line, which ſeperates the Light from the Dark Parts on her Surface is not even or regular, but tooth'd and jagg'd with innumerable Breaks; and even in the dark Parts, near the Borders of the lucid Surface, there are ſeen ſome ſmall Spots enlighten'd by the Sun, which are very viſible when the ☽ is three or four Days old, and which can be nothing elſe but the Tops of *Mountains* or *Rocks*; ſince it is impoſſible for the Sun's Rays to fall upon thoſe Parts only, unleſs they were higher t' an the Reſt of the Surface.

The *Lunar Mountains* are found to be *higher*, in Proportion to the Body of the ☽ than any Hills upon our Globe.—The Manner of calculating their Heights is this.



Let EGD be the Surface of the ☽ and ECD the Diameter of the Circle bounding *Light* and *Darkneſs*. A the Top of a Hill within the dark Part, when it firſt begins to be illuminated by a Ray of Light coming from the Sun at S. Then obſerve with a Teleſcope the Proportion of the Right Line AE, (*i. e.* the Diſtance of the Point A from the lucid Part) to the Diameter (or Semidiameter) of the ☽ ED, for that being aſcertain'd, you have in the Triangle AEC, Right Angled at E, (where the Ray of Light touches the ☽) the two Sides AE and CE, to find the *Hypothenuſe* AC, from which ſubtracting BC = EC, there will remain AB the Height of the *Mountain*.

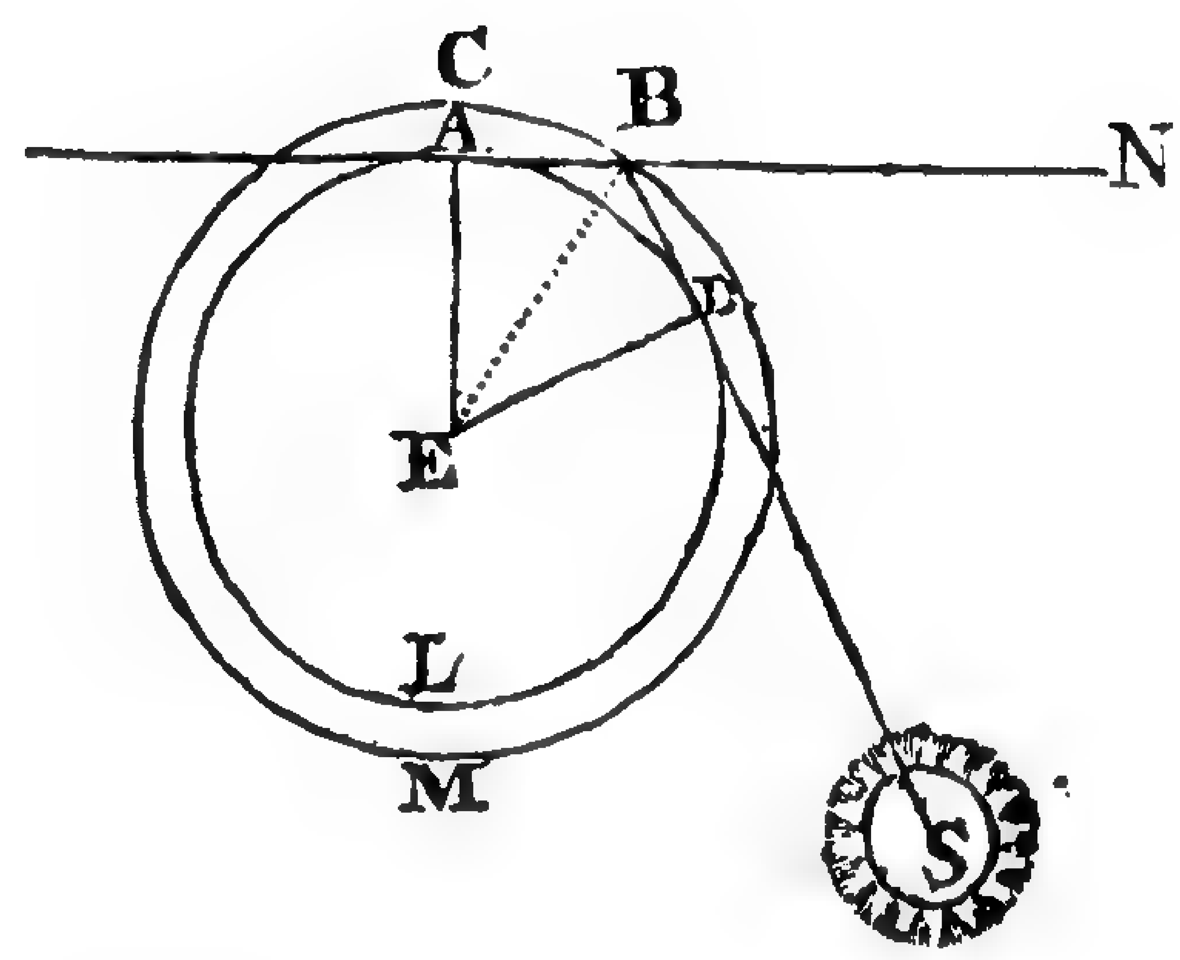
Ricciolus, on viewing the ☽ when about four Days old, obſerv'd the Top of a Hill called *Saint Catherine*, near the N. Part of *Mount Taurus*, (ſee my *Aſtronomy*) to be illuminated, and that it was then diſtant from the Surface about $\frac{1}{8}$ of the Moon's *Semidiameter*. Now as the *Semidiameter* of the ☽ ECis about 1088 Miles, the Line AE being $\frac{1}{8}$ of it, muſt be = 136 Miles. Conſequently, if the □ of EC and □ of AE be added together, and then the □ Root of it be extracted, it will give the Line AC, from which ſubtracting the Moon's *Semidiameter* BC or CE, the Remainder, which is 8 Miles will be the Height of the *Mountain* ſought.

PROBLEM

PROBLEM XVIII.

To meaſure the *Height* of the *Atmoſphere*.

The *Atmoſphere* is that Circle of *vaporous Air* ſurrounding the Earth, which being illuminated by the *Sun*'s Rays makes the Brightneſs and Glory of the *Firmament* we behold, whilſt the Sun continues above the *Horizon*. And, after the Sun is gone down, the *Atmoſphere*, which is higher than we are, will ſtill continue to be illuminated by thoſe Rays paſſing by the Earth over our Heads; but this *Brightneſs* grows *leſs* and *leſs* as the Sun deſcends lower, till he arrives at 18° below the *Horizon*; when all the Parts of the Air above fall out of his Rays, and, conſequently, become dark.



To make this plainer; ſuppoſe the inner Circle ADL repreſents the Earth, and the Circle CBM the *Atmoſphere*. Suppoſe a Perſon ſtanding upon the Earth at A whoſe *ſenſible Horizon* is AN. Alſo let SB be a Ray of Light coming from the *Sun*, touching the Earth at D, which falls upon the diſtant Part of the Air, in the Horizon at B, at which time *Twilight* ceaſes. This has been found to happen when the Sun is deſcended 18° below the *weſtern* Horizon in the *Evening*; and alſo when he is approached within 18° of the *eaſtern* Horizon in the *Morning*.

Now as the Arch AD is 18°, we have, by drawing the Line EB, two equal Triangles, from either of which we may find the Height of the *Atmoſphere* requir'd.——For in the Triangle ABE, Right Angled at A, we have given the Side AE the *Earth*'s *Semidiameter*, and the Angle AEB = 9° = half the Arch of the *Sun*'s Deſcent below the *Horizon*, and the Angle ABE = 81°, to find the *Hypothenuſe* EB, from which if you ſubtract the *Semidiameter* of the Earth, the Remainder will be the Height of the *Atmoſphere*.

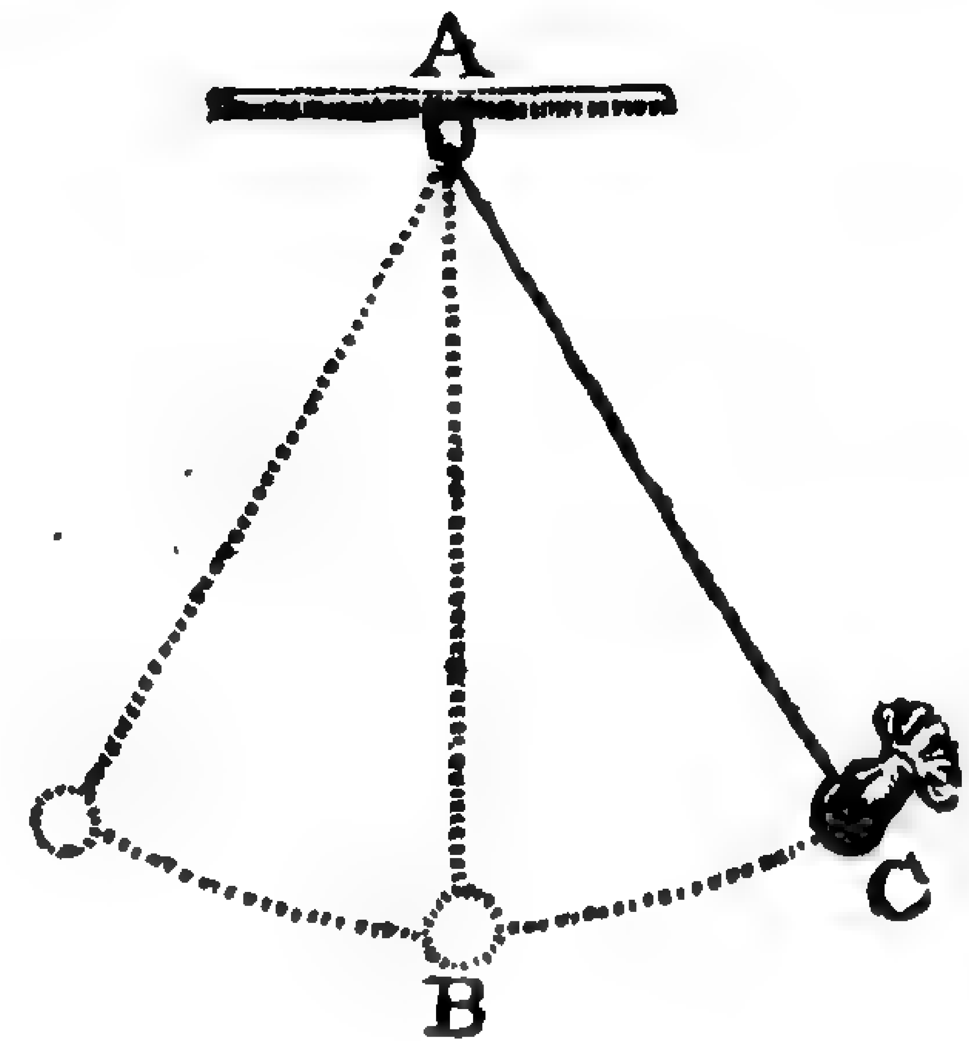
Thus, Ang. B : Earth's Semid^r. AE :: N. Rad. : Hypoth. EB
As 81 ——— 4000 ——— 81.99 —— 4048
4000 Earth's Semidiameter ſubtract
48 Height of the Atmoſphere requir'd.

PROBLEM

PROBLEM XIX.

To measure the *Distance* of a *Cloud*, from which issues *Lightnings* and *Thunder*.

Take a small *Ball* of *Lead*, *Ivory*, or any other matter, and affix it to the End of a fine *Thread*. Then measure from the Center of the Ball, along the Thread, exactly 39.2 *Inches*, where make a *Loop*. This done, suspend it by that Loop to the Ceiling of the Room, or to any other Place where it may hang freely, and vibrate backwards and forwards like a *Pendulum*, as in this *Figure*. Now the Property of this little Instrument is, that *each* Vibration, whether it passes through a larger or a smaller Space, will be performed in *one Second* of Time.



Being thus prepar'd, take the Ball in your Hand, and drawing it aside from its perpendicular Direction AB, to any Distance, suppose to C, hold it there till you see the flash of *Lightning* pass by, at which Moment let it go, and count the *Number of Vibrations* till you hear the Stroke of the *Thunder*. Then these Vibrations multiplied by 1142, (the Number of Feet, *Sound* uniformly passes thro' in each *Second)* the Product will be the Height of the *Cloud* in Feet, if it *be nearly over the Place where you are*; or its Distance from you in any other *Situation*.

Thus, suppose the *String* is found to make 8 Vibrations between the *Lightning* and the *Thunder*; then $8 \times 1142 = 9136$ Feet, which, divided by 5280 (the Feet in 1 Mile) gives $1\frac{3}{4}$ Mile nearly; and so far is that alarming Tempest from you.

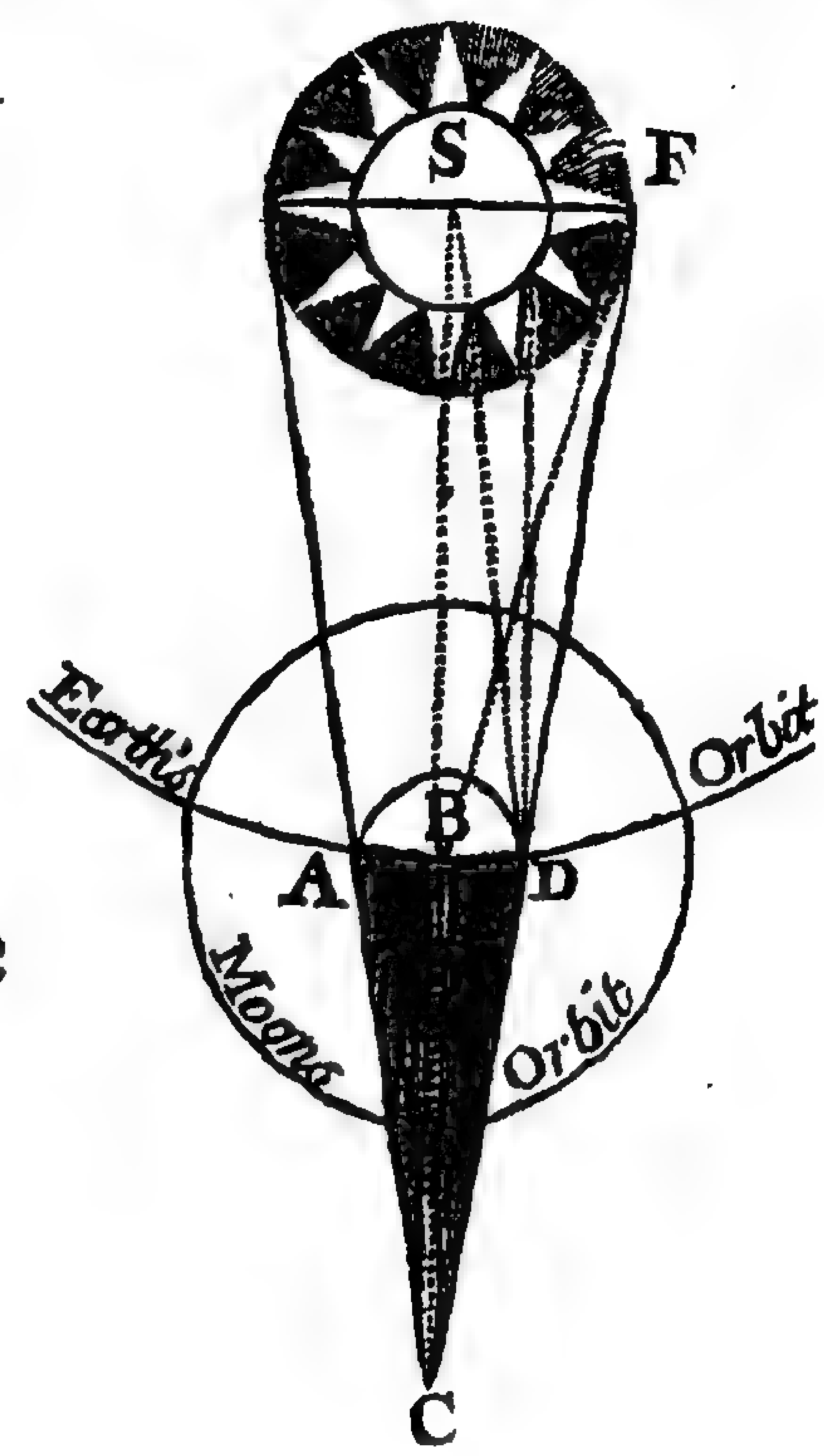
In this Manner you may continue to measure the Distance of the *Cloud* all the Time it passes from your *Zenith* to the *Horizon*, and by that Means be acquainted with the *Danger* it seems to threaten the *Neighbourhood*, as well as the *Extent* of the *visible Hemisphere* of *Clouds*.

The *Distance* also of a *Ship* at *Sea*, or a *Fort*, may be estimated in the same Manner, by counting the Vibrations from the *Flash of the Powder* to *the Report of the Gun*.

PROBLEM XX.

To calculate the *Length* of the *Earth*'s or *Moon*'s *Shadow*.

The Angle of the *Cone* ACD of the *Earth*'s *Shadow* in the annex'd Figure, is equal to the *Sun*'s apparent *Diameter* *, which, at a mean Diſtance from us, is about 32 *Minutes*.—Hence, in the Triangle ACB, Right Angled at B, we have the Angle ACB = 16 *Minutes*, the apparent *Semidiameter* of the *Sun*, and AB the *Semidiameter* of the *Earth* = 1; to find AC or BC the Length of the *Shadow*, which is done thus.



	∠ ACB	:	Side AB	::	N. Rad.
As	.266	—	1	—	57.3
					1

.266)57.300(215 Semidiameters = AC
532
―――
410
266
―――
1440
1330
―――
110

Thus, when the *Sun* is at a *mean* Diſtance from us, the *Shadow* of the *Earth* reaches about 215 *Semidiameters* beyond it: But when the *Sun* is at his *greateſt* or *leaſt* Diſtance, the *Shadow* will be *lengthened* or *ſhortened* 3 or 4 *Semidiameters*, more or leſs.

Hence, you may alſo determine the *Height* of the *Moon*'s *Shadow*: For, as the *Moon* is never at any great Diſtance from the *Earth*, the apparent *Semidiameter* of the *Sun* muſt be nearly the ſame *there* as *here*. Conſequently, the *Moon*'s *Shadow* muſt contain the ſame Number of *Semidiameters* of the *Moon*, as the *Earth*'s *Shadow* does *Semidiameters* of the *Earth*: Which *Semidiameters*, multiply'd by the *Miles* in the *Semidiameter* of the *Moon* or *Earth*, will give the Length of the *Shadow* reſpectively in Miles.

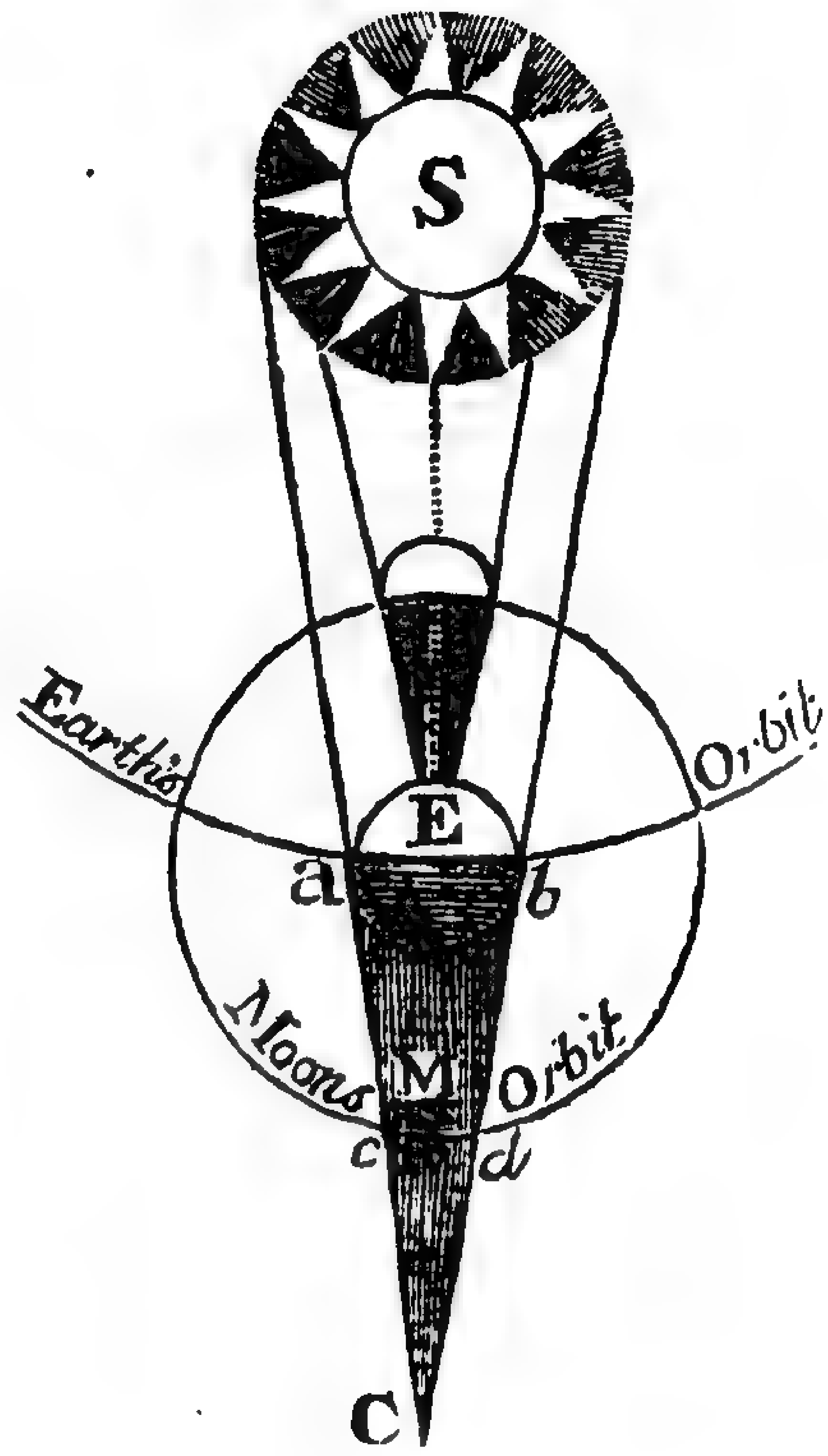
* The *Semiangle* of the *Cone* of the *Earth*'s *Shadow* BCD, is equal to the apparent *Semidiameter* of the *Sun* view'd from the Top of the Shadow, which Angle is always equal (in the Shadow of every Planet) to the apparent *Semidiameter* of the *Sun* SBF, leſſen'd by his *Horizontal Parallax* BSD at that Planet. But as the *Horizontal Parallax* of the *Sun*, i. e. the Angle under which the *Earth* is ſeen from thence, is ſcarcely 10 *Seconds*, it may be omitted, as is done in the above Calculation.

PROBLEM

PROBLEM XXI.

To calculate the *Diameter* of the *Earth*'s *Shadow* at the Diſtance of the *Moon*; and alſo, the *Diameter* of the *Moon*'s *Shadow* at the *Earth*.

In the following Figure let S repreſent the *Sun*, E the *Center* of the *Earth*, M the *Moon*, EC the *Cone* of the *Earth*'s *Shadow* (at a mean) = 215 *Semidiameters* of the *Earth*: Then MC will be the *Cone* of the *Earth*'s *Shadow* reaching beyond the *Moon*, whoſe Length is thus found.



	Semid^rs.
From EC the Cone of the Earth's Shadow	= 215
Subtract EM the Diſt. of the Moon in Earth's Semid.	= 60*
Remains MC the Shad. of the Earth beyond the Moon	= 155

Then, by Reaſon of ſimilar Triangles, it will always hold;

As the Length of the whole Shadow	EC
Is to the Diameter of the Earth	ab
So is the Length of the Shadow beyond the Moon	MC
To the Diameter of the Shadow at the Moon	cd.

```
    EC   :   ab   ::   MC
As 215  —   7964  —   155
             155
           -----
           39820
          39820
          7964
          -------
215)1234420(5741 Miles = cd, the Diameter of the Earth's Shadow at the Diſtance of the Moon.
    1075
    ----
     1594
     1505
     ----
       892
       860
       ---
        320
        215
        ---
        105
```

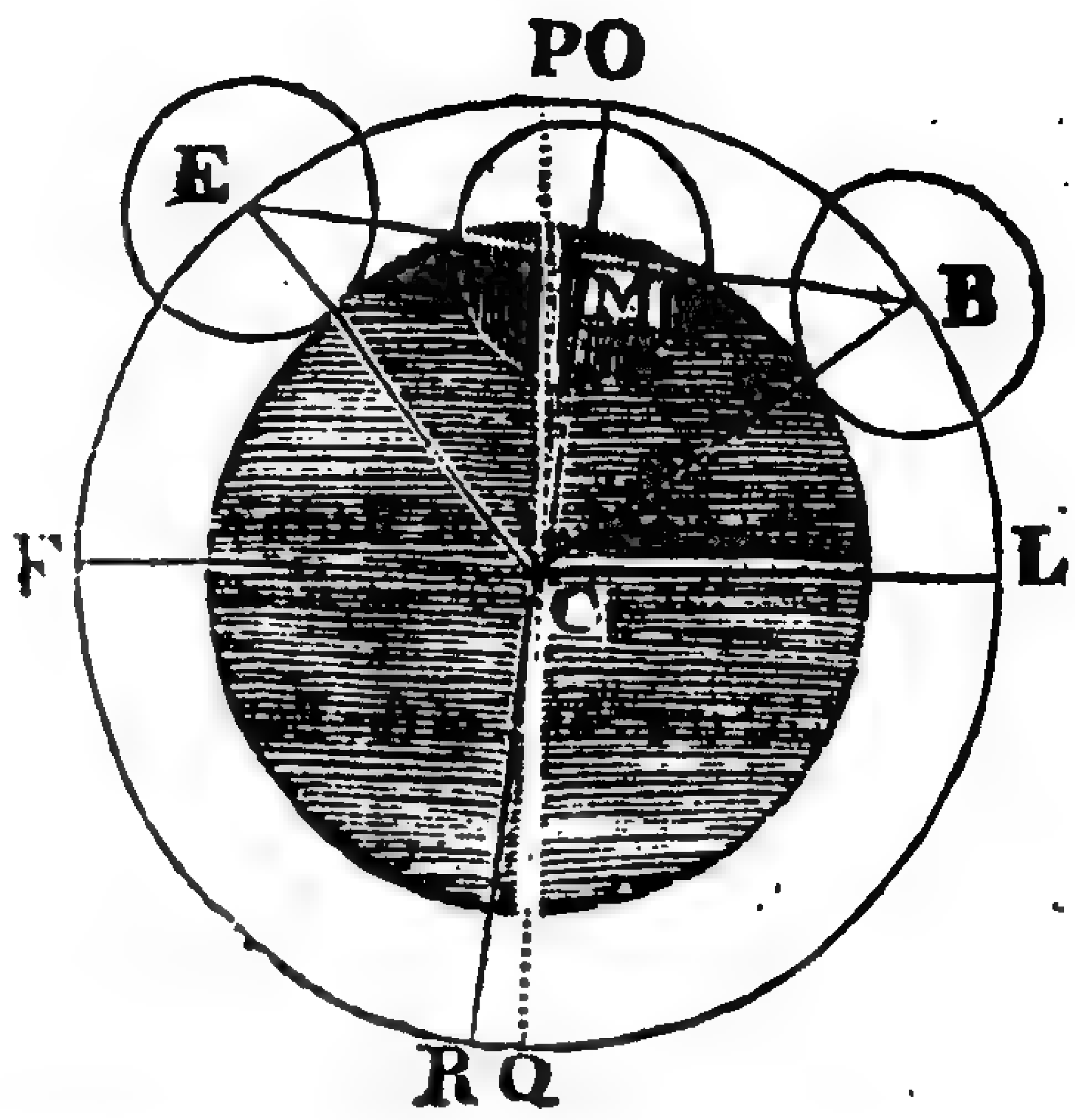
By this Problem alſo may be found the *Diameter* of the *Moon*'s *Shadow* at the Surface of the Earth, and, conſequently, how much of the Earth is involv'd in that *Shadow* in an *Eclipſe* of the *Sun*. For the *Length* of the *Moon*'s *Shadow* is found to be about 60 *Semidiameters* of the Earth, which is nearly the *Moon*'s *mean* Diſtance from us; her *Shadow*, in that State, muſt, therefore, reach as far as the Center of the Earth.—But as the *Moon* is, ſometimes, almoſt 4 *Semidiameters* of the Earth *nearer*, the *Shadow* muſt reach 4 *Semidiameters* beyond the Center of the Earth; and when the *Moon* (as ſhe ſometimes is) is 4 *Semidiameters further* from us, the *Shadow* will then not reach the Earth at all. In ſuch Caſe, the *Sun*, though centrally *eclips'd*, will not be *totally* covered by the *Moon*; but an *Annulus*, or Ring of Light, will appear round the Border of that Luminary, as happen'd April 1ſt, 1764.

* The Method of finding the *Diſtance* of the *Moon*, in Semidiameters of the Earth, is ſhewn at Problem XVI.

PROBLEM XXII.

To calculate the *Beginning*, *End*, and *total Duration* of an *Eclipse*.

Having, from *Astronomical Tables*, obtain'd the *Time* of the *Middle of the Eclipse*, which suppose to be December 21 Day, 11 Hours, 49 Minutes, with the *Latitude* of the *Moon* at that Time, = 40 Minutes, you may then proceed to find the *Beginning*, *End*, and *total Duration*, as follows.



From a Scale of equal Parts, of any Size, take off the *Semidiameter* of the the *Earth*'s Shadow, which, at the Distance of the ☽ (at a Mean) is about 42′; and, setting one Foot in C, describe the inner *shaded Circle*, to express that Part of the Cone of the Earth's Shadow cut off at that Place where the ☽ passes thro' in that *Eclipse*.—With the *Sum* of the *Semidiameter* of the ☽ = 16′, and *Earth*'s Shadow = 42′, (which together = 58′) taken from the same equal Parts describe the outer Circle.—Draw the Line FL through the Center, to represent the *Ecliptic*, or Path of the *Earth*'s Shadow; cross it, at Right Angles, with the dotted Line PQ, to express the *Poles* of the *Ecliptic*.—Then with a *Line of Chords*, or a *Protractor*, set off $5\frac{1}{2}°$ from P, upon the outer Circle, towards the *Right Hand*, because the *Latitude* of the ☽ is *North ascending*, to express the Angle of the *Moon*'s *Path* with the *Ecliptic*, and draw the Line OR.—Take the *Latitude* of the ☽ = 40′, from the same Scale of equal Parts, and set it from C to M upon the Line CO.—Then draw a Line through M, at Right Angles to CO, and that Line will represent the *Path* of the ☽ during the *Eclipse*.—Next, with the *Semidiameter* of the ☽ = 16′, taken from the equal Parts, describe, on the three Points B, M, and E, severally, the three *little Circles*; so will the Circle at B represent the ☽ at the *Biginning*, that at M the *Middle*, and that at E, the *End* of the Eclipse.

Now, from the Center C, draw two Lines to B and E; then in the *Right Angled Triangle* CMB, Right Angled at M, we have given CM the *Latitude* of the ☽ = 40, and CB = CE the *Sum* of the *Semidiameters* of the *Moon* and and *Earth*'s *Shadow* = 58; to find MB = ME, the *Motion* of *Half Duration* of the *Eclipse*.

From Square of 58 = 3364
Take Square of 40 = 1600

Extract the Root 1764(42 *Minutes*, = the Motion of *Half* the *Duration*.
16
82)164
164

And because the Moon passes over 31 of these Minutes (at a mean Rate) in one Hour, we have this Proportion.——If 31 Min. : 1 Hour :: 42 Min. : 1 Hour 21 Min. which *subtracted from*, and *added to*, the Middle, will give the *Beginning* and *End*.

		D.	H.	M.
Middle of the Eclipse	Dec.	21	11	49
Half Duration subtract and add			1	21
Beginning		21	10	28
End		21	13	10
Total Duration			2	42

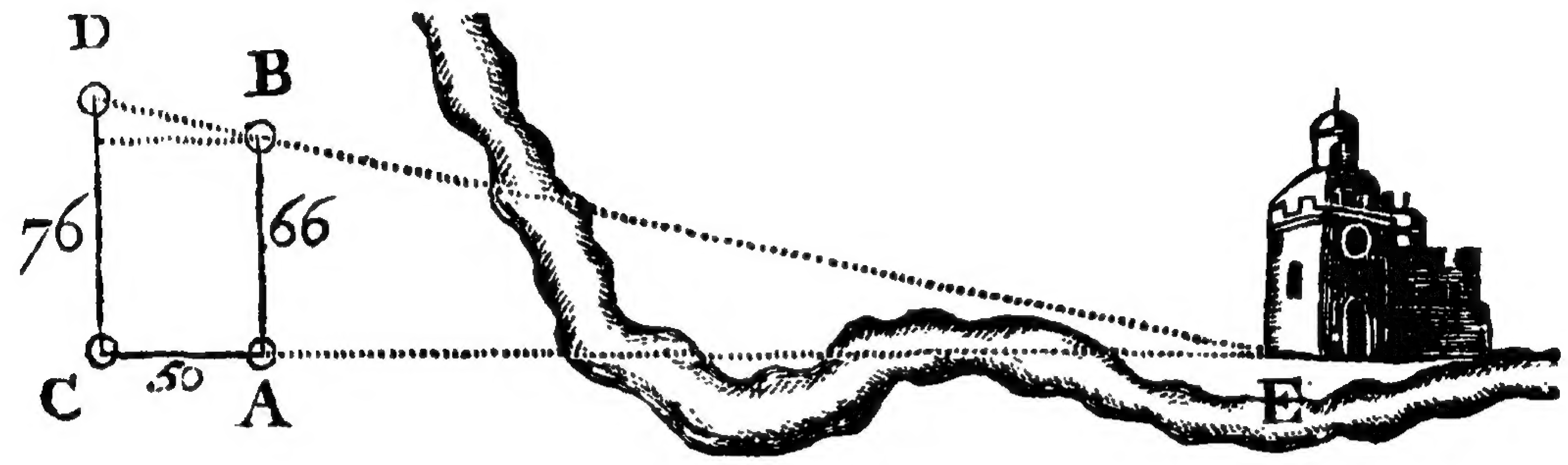
Note. The Middle of an *Eclipse*, with the *Lat.* of the *Moon*, may be easily had from my *Astronomy*.

PROBLEM

PROBLEM XXIII.

To take the *Diſtance* of an *inacceſſible Object* without the Help of *any Inſtrument.*

Suppoſe E, in the following Figure, to be a *Fort,* whoſe *Diſtance* you want to know, and you cannot approach it, on Account of ſome *Moat, Ditch,* or *River,* lying between you and the *Object.*



Firſt, at ſome Diſtance from the *Ditch* or *River,* ſet up a *Stick,* as at C; then advance, in a Right Line, towards E, any Number of Yards, ſuppoſe 50, and ſet up another Stick at A; next, move, in a Line *perpendicular* to CE, from A to B, any Diſtance, ſuppoſe 66 Yards, and ſet up another Stick at B; then return back to C, where you began, and remove from thence, in a Line *perpendicular* to CE, till you ſee the Stick at B and the Object E in a Right Line, and ſet up another Stick in that Place at D, meaſuring the Diſtance from C to D, which ſuppoſe 76 Yards. Then it will always hold;—

$$\left.\begin{array}{lcr} \text{As the Difference between AB and CD} & = & 10 \\ \text{Is to the Length between A and C} & = & 50 \\ \text{So is the Diſtance CD} & = & 76 \\ \text{To the Diſtance between C and E} & = & 380 \end{array}\right\} \text{Yards.}$$

NOTE. If, in the *third Term,* you had us'd the *Diſtance* AB = 66 Yards, you would have obtain'd the Diſtance from A to E = 330.

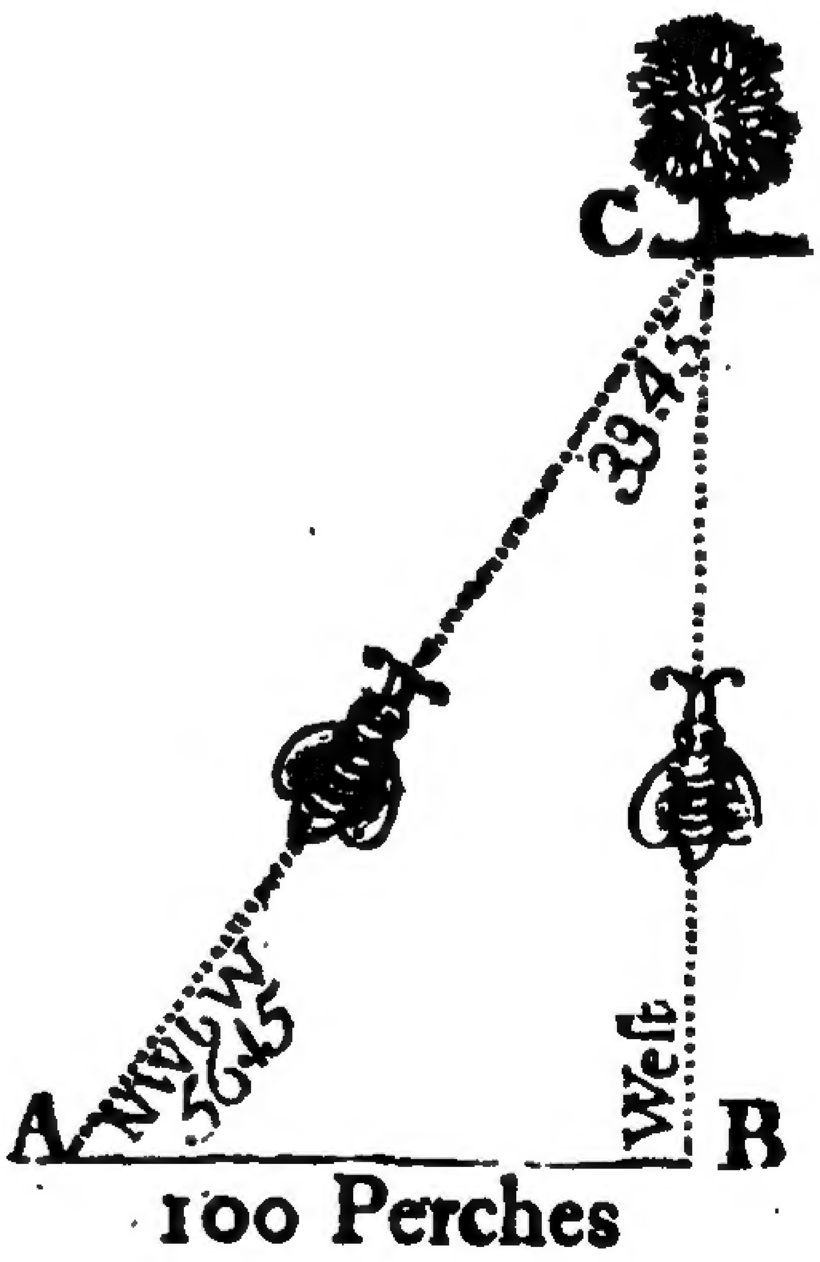
PROBLEM XXIV.

To find, by a *new Method*, where the *Bees* hive in large and extenſive *Woods*, in Order to obtain their *Honey*.

Take a *Plate* or ſmall Piece of *Board*, on which is ſpread a little *Honey* or *Treacle*, and ſet it down on a *Rock* or Stump of a *Tree* within the *Wood*. This the *Bees* will ſoon find out if any are near, for it is generally believ'd they ſmell Things of that Nature at the Diſtance of a Mile, or further. Whilſt theſe little *Creatures* are feeding, ſecure *two* or *three* of them in a Box, or ſomething convenient. Then let one of them go, obſerving carefully, by a *Pocket Compaſs*, the Courſe he takes; for *Bees*, after they riſe in the *Air*, fly directly in a ſtrait Line to the *Tree* where their Hive is.

Suppoſe, for Example, the *firſt Bee* is found to fly directly *Weſt*; then you may be ſure the *Tree* is ſome where in that Line from your preſent Station. But, in order to know how far, you muſt make an *Offſet*, either *North* or *South*, as large as you can, which in this caſe we will ſuppoſe to be 100 *Rods* or *Perches* (the larger the better) to the *South*. Here you muſt let go another *Bee*, obſerving his Courſe as before, (for this Bee, being loaded like the other, will fly *directly* to the *Hive*) which Courſe we will ſuppoſe to be N. W. by W. —56° 15′ towards the *Weſt*, it only remains now to find where theſe two Courſes or Lines interſect or meet with each other, for there you will find the Tree in which the *Honey* is.

This may be eaſily done.—For in the *Right Angled Triangle* ABC, are given the Right Angle B, the *Courſe* of the firſt Bee, the Angle at A, the *Courſe* of the ſecond Bee, and the Diſtance AB; to find BC, or AC, the Diſtance of the *Tree* from either Station.



	∠C	:	Baſe	::	N. Rad.	:	Diſt. AC
As	33·75	—	100	—	60·7	——	179·8 Perches.

Formerly, they found the *Honey* by ſurprizing the Bees, and following them, one after another, till they found out the Hive; but ſince this *Trigonometrical Method* has been us'd, the Searchers diſcover that Booty in a few Hours, which before requir'd many Days.

CONCLUSION.

CONCLUSION.

THESE few *Problems* are sufficient to point out the great *Use* of this Branch of *Learning*. The Advantages resulting from it to Society are very great;—almost infinite.——Nothing however posited in the *Heavens*;—nothing upon the *Earth* or *Seas*;—but its *Distance* and *Dimensions* may be ascertained by it.——It is no Wonder then, that *Pythagoras*, a learned Philosopher of *Samos*, when he had discover'd that famous *Proposition* (47th of 1st Book of *Euclid*) which is the *Foundation* of this *Science*, should, in Gratitude, sacrifice an *Hecatomb*, i. e. 100 Oxen, to the *Muses*, for inspiring him with such an useful Invention, which he judg'd beyond the Power of *human Abilities* to discover.

Thus by one plain *Geometrical Figure*, having three Sides and three Angles, and assisted by the *Rule of Three*, you see what amazing *Truths* may be discover'd. This illustrates not only the old Motto,—*Tria sunt omnia*—but also proves the *Truth* of Mine in the *Title-page*.

Cuncta Trigonus *habet, satagit quæ docta* Mathesis,
Ille *aperit clausum quicquid* Olympus *habet*.

Which may be English'd thus;

In Heaven *the* latent Science *lay conceal'd*
Till the Triangle *came, and* Truth *reveal'd*.

FINIS.

Just Publiſh'd, by the ſame AUTHOR.

A *View of the Earth*; being a Short but Comprehenſive Syſtem of *Modern Geography*: Illuſtrated with a *new Sett* of *Maps*, &c. and a curious *Geographical Clock*, which points the *Hour* at any *Kingdom* or *Nation* upon the Earth at one View.

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